

Name: KEY
Geometry Regents Exam Review #1

Date: _____
Period: _____

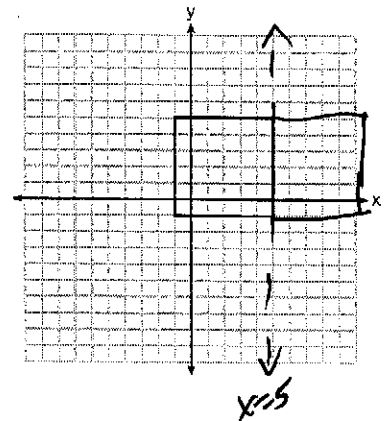
1. A parallelogram must be a rectangle when its
- 1) diagonals are perpendicular
 - 2) diagonals are congruent
 - 3) opposite sides are parallel
 - 4) opposite sides are congruent
2. If $\triangle A'B'C'$ is the image of $\triangle ABC$, under which transformation will the triangles *not* be congruent?
- 1) reflection over the x -axis
 - 2) translation to the left 5 and down 4
 - 3) dilation centered at the origin with scale factor 2
 - 4) rotation of 270° counterclockwise about the origin

3. If the rectangle below is continuously rotated about side w , which solid figure is formed?
- 1) pyramid
 - 2) rectangular prism
 - 3) cone
 - 4) cylinder



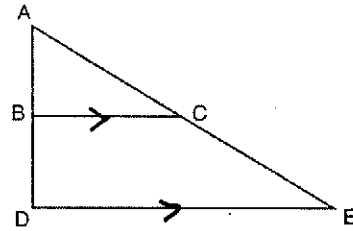
4. Which expression is always equivalent to $\sin x$ when $0^\circ < x < 90^\circ$?
- 1) $\cos(90^\circ - x)$
 - 2) $\cos(45^\circ - x)$
 - 3) $\cos(2x)$
 - 4) $\cos x$

5. In the diagram below, a square is graphed in the coordinate plane. A reflection over which line does *not* carry the square onto itself?
- 1) $x = 5$
 - 2) $y = 2$
 - 3) $y = x$
 - 4) $x + y = 4$



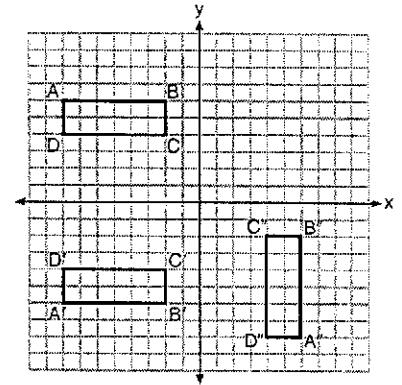
6. The image of $\triangle ABC$ after a dilation of scale factor k centered at point A is $\triangle ADE$, as shown in the diagram below. Which statement is always true?

- 1) $\frac{2AB}{AD} = \frac{AD}{DE}$
- 2) $AD \perp DE$
- 3) $\frac{AC}{CE} = \frac{CE}{DE}$
- 4) $BC \parallel DE$



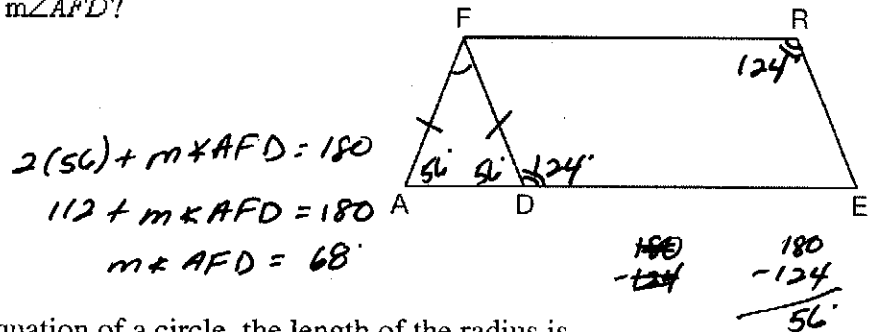
7. A sequence of transformations maps rectangle $ABCD$ onto rectangle $A''B''C''D''$, as shown in the diagram below. Which sequence of transformations maps $ABCD$ onto $A'B'C'D'$ and then maps $A'B'C'D'$ onto $A''B''C''D''$?

- 1) a reflection followed by a rotation
- 2) a reflection followed by a translation
- 3) a translation followed by a rotation
- 4) a translation followed by a reflection



8. In the diagram of parallelogram $FRED$ shown below, \overline{ED} is extended to A , and \overline{AF} is drawn such that $\overline{AF} \cong \overline{DF}$. If $m\angle R = 124^\circ$, what is $m\angle AFD$?

- 1) 124°
- 2) 112°
- 3) 68°
- 4) 56°



9. If $x^2 + 4x + y^2 - 6y - 12 = 0$ is the equation of a circle, the length of the radius is

- 1) 25
- 2) 16
- 3) 5
- 4) 4

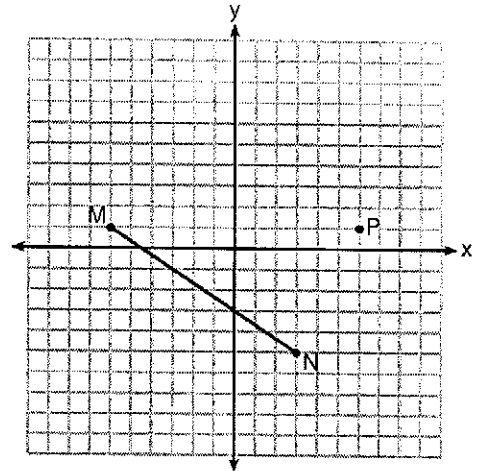
$$x^2 + 4x + 4 + y^2 - 6y + 9 = 12 + 4 + 9$$

$$(x + 2)^2 + (y - 3)^2 = 25$$

$$r^2 = 25$$

$$r = 5$$

10. Given \overline{MN} shown below, with $M(-6, 1)$ and $N(3, -5)$, what is an equation of the line that passes through point $P(6, 1)$ and is parallel to \overline{MN} ?



- 1) $y = -\frac{2}{3}x + 5$
- 2) $y = -\frac{2}{3}x - 3$
- 3) $y = \frac{3}{2}x + 7$
- 4) $y = \frac{3}{2}x - 8$

$$m_{\overline{MN}} = \frac{-5-1}{3-(-6)} = \frac{-6}{9} = -\frac{2}{3}$$

$$m = -\frac{2}{3} \quad P(6, 1)$$

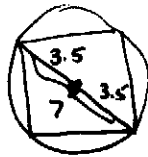
$$y - 1 = -\frac{2}{3}(x - 6)$$

$$y - 1 = -\frac{2}{3}x + 4$$

$$y = -\frac{2}{3}x + 5$$

11. Linda is designing a circular piece of stained glass with a diameter of 7 inches. She is going to sketch a square inside the circular region. To the nearest tenth of an inch, the largest possible length of a side of the square is

- 1) 3.5
- 2) 4.9
- 3) 5.0
- 4) 6.9



$$x^2 + x^2 = 7^2$$

$$2x^2 = 49$$

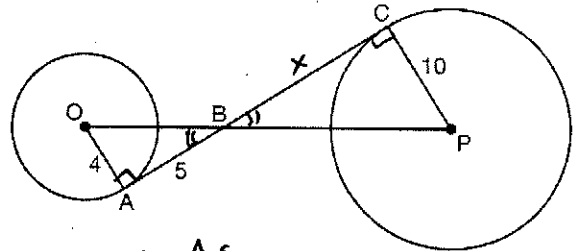
$$x^2 = \frac{49}{2}$$

$$x = \sqrt{\frac{49}{2}}$$

$$x \approx 4.9$$

12. In the diagram shown below, \overline{AC} is tangent to circle O at A and to circle P at C , \overline{OP} intersects \overline{AC} at B , $OA = 4$, $AB = 5$, and $PC = 10$. What is the length of \overline{BC} ?

- 1) 6.4
- 2) 8
- 3) 12.5
- 4) 16



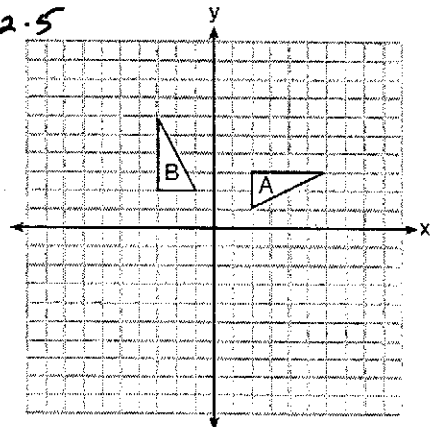
$$\frac{4}{10} = \frac{5}{x} \quad \sim \Delta s$$

$$4x = 50$$

$$x = 12.5$$

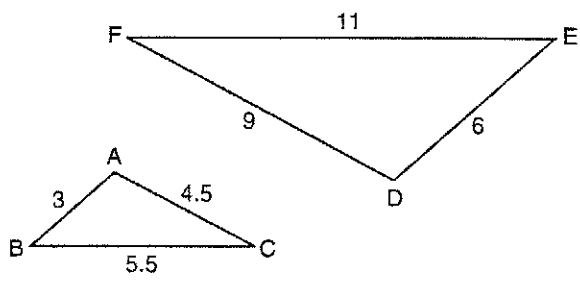
13. In the diagram below, which single transformation was used to map triangle A onto triangle B?

- 1) line reflection
- 2) rotation
- 3) dilation
- 4) translation



14. In the diagram below, $\triangle DEF$ is the image of $\triangle ABC$ after a clockwise rotation of 180° and a dilation where $AB = 3$, $BC = 5.5$, $AC = 4.5$, $DE = 6$, $FD = 9$, and $EF = 11$. Which relationship must always be true?

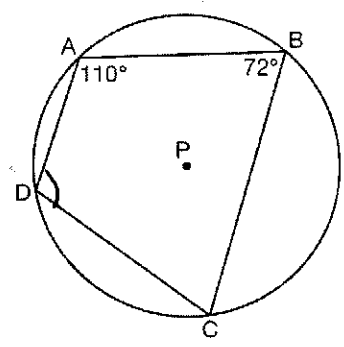
- 1) $\frac{m\angle A}{m\angle D} = \frac{1}{2}$
- 2) $\frac{m\angle C}{m\angle F} = \frac{2}{1}$
- 3) $\frac{m\angle A}{m\angle C} = \frac{m\angle F}{m\angle D}$
- 4) $\frac{m\angle B}{m\angle E} = \frac{m\angle C}{m\angle F}$



15. In the diagram below, quadrilateral $ABCD$ is inscribed in circle P . What is $m\angle ADC$?

- 1) 70°
- 2) 72°
- 3) 108°
- 4) 110°

$72 + m\angle D = 180$
 $m\angle D = 108$



16. A hemispherical tank is filled with water and has a diameter of 10 feet. If water weighs 62.4 pounds per cubic foot, what is the total weight of the water in a full tank, to the nearest pound?

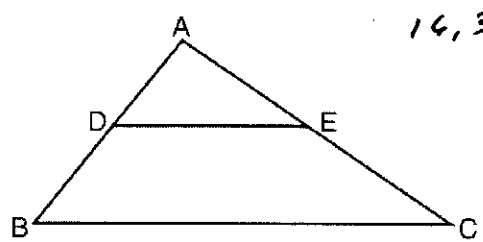
- 1) 16,336
- 2) 32,673
- 3) 130,690
- 4) 261,381



$V = \frac{4}{3}\pi r^3$
 $V = \frac{4}{3}\pi(5^3)$
 $V = \frac{523.5987756}{2}$
 $V = 261.7993878$
 $\cdot 62.4$
 $16,336.2818$
 165

17. In the diagram below, $\triangle ABC \sim \triangle ADE$. Which measurements are justified by this similarity?

- 1) $AD = 3$, $AB = 6$, $AE = 4$, and $AC = 12$
- 2) $AD = 5$, $AB = 8$, $AE = 7$, and $AC = 10$
- 3) $AD = 3$, $AB = 9$, $AE = 5$, and $AC = 10$
- 4) $AD = 2$, $AB = 6$, $AE = 5$, and $AC = 15$



$\frac{3}{6} \neq \frac{4}{12}$ $\frac{3}{4} \neq \frac{5}{10}$
 $\frac{5}{8} \neq \frac{7}{10}$ $\frac{2}{6} = \frac{5}{15}$ ✓ $\frac{AD}{AB} = \frac{AE}{AC}$

18. Triangle FGH is inscribed in circle O , the length of radius \overline{OH} is 6, and $\overline{FH} \cong \overline{OG}$.
 What is the area of the sector formed by angle FOH ?

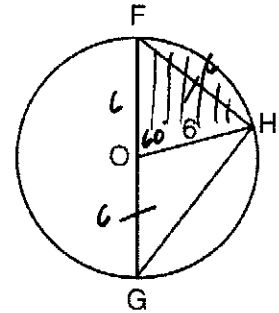
- 1) 2π
- 2) $\frac{3}{2}\pi$
- 3) 6π
- 4) 24π

$$\frac{A}{\pi r^2} = \frac{60}{360}$$

$$\frac{A}{36\pi} = \frac{1}{6}$$

$$6A = 36\pi$$

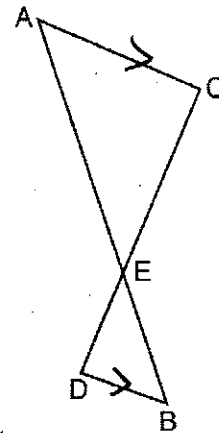
$$A = 6\pi$$



19. As shown in the diagram below, \overline{AB} and \overline{CD} intersect at E , and $\overline{AC} \parallel \overline{BD}$.

Given $\triangle AEC \sim \triangle BED$, which equation is true?

- 1) $\frac{CE}{DE} = \frac{EB}{EA}$
- 2) $\frac{AE}{BE} = \frac{AC}{BD}$
- 3) $\frac{EC}{AE} = \frac{BE}{ED}$
- 4) $\frac{ED}{EC} = \frac{AC}{BD}$



20. A triangle is dilated by a scale factor of 3 with the center of dilation at the origin. Which statement is true?

- 1) The area of the image is nine times the area of the original triangle.
- 2) The perimeter of the image is nine times the perimeter of the original triangle.
- 3) The slope of any side of the image is three times the slope of the corresponding side of the original triangle.
- 4) The measure of each angle in the image is three times the measure of the corresponding angle of the original triangle.

21. The Great Pyramid of Giza was constructed as a regular pyramid with a square base. It was built with an approximate volume of 2,592,276 cubic meters and a height of 146.5 meters. What was the length of one side of its base, to the nearest meter?

- 1) 73
- 2) 77
- 3) 133
- 4) 230



$$V = \frac{1}{3} B h$$

$$2,592,276 = \frac{1}{3} s^2 (146.5)$$

$$2,592,276 = 48.83333333 s^2$$

$$\frac{2,592,276}{48.83333333} = \frac{48.83333333 s^2}{48.83333333}$$

$$\sqrt{s^2} = \sqrt{53,084.15017}$$

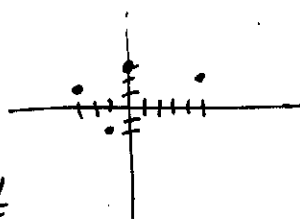
$$s = 230.3999787$$

22. A quadrilateral has vertices with coordinates $A(-3, 1)$, $B(0, 3)$, $C(5, 2)$, and $D(-1, -2)$. Which type of quadrilateral is this?

- 1) rhombus
- 2) rectangle
- 3) square
- 4) trapezoid

$$m_{AB} = \frac{3-1}{0-(-3)} = \frac{2}{3}$$

$$m_{BC} = \frac{2-3}{5-0} = -\frac{1}{5}$$



$$m_{CD} = \frac{-2-2}{-1-5} = \frac{-4}{-6} = +\frac{2}{3}$$

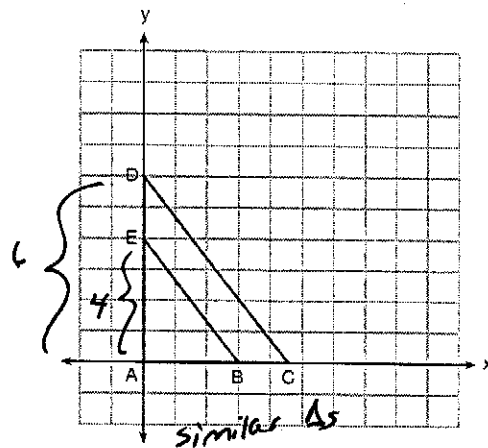
$$m_{AD} = \frac{-2-1}{-1+3} = \frac{-3}{2}$$

23. In the diagram below, $\triangle ABE$ is the image of $\triangle ACD$ after a dilation centered at the origin. The coordinates of the vertices are $A(0, 0)$, $B(3, 0)$, $C(4.5, 0)$, $D(0, 6)$, and $E(0, 4)$.

The ratio of the lengths of \overline{BE} to \overline{CD} is

- 1) $\frac{2}{3}$
- 2) $\frac{3}{2}$
- 3) $\frac{3}{4}$
- 4) $\frac{4}{3}$

$$\frac{4}{6} = \frac{2}{3}$$



24. Line $y = 3x - 1$ is transformed by a dilation with a scale factor of 2 and centered at $(3, 8)$. The line's image is

- 1) $y = 3x - 8$
- 2) $y = 3x - 4$
- 3) $y = 3x - 2$
- 4) $y = 3x - 1$

$$8 = 3(3) - 1$$

$$8 = 9 - 1$$

$$8 = 8 \checkmark$$

point is on line
so the image is the same line

Show your work!

25. A wooden cube has an edge length of 6 centimeters and a mass of 137.8 grams. Determine the density of the cube, to the nearest thousandth. State which type of wood the cube is made of, using the density table below.



$$d = \frac{m}{v}$$

$$d = \frac{137.8 \text{ g}}{216 \text{ cm}^3}$$

$$d = 0.637962963$$

$$V = s^3$$

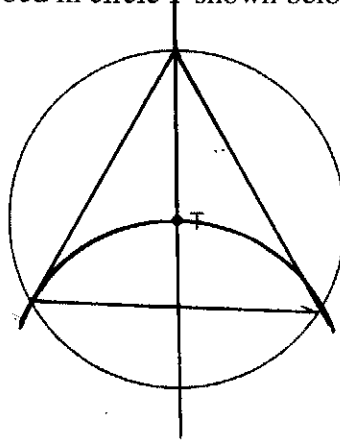
$$V = 6^3$$

$$V = 216$$

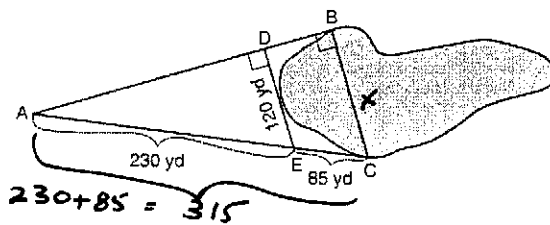
Ash

Type of Wood	Density (g/cm ³)
Pine	0.373
Hemlock	0.431
Elm	0.554
Birch	0.601
Ash	0.638
Maple	0.676
Oak	0.711

26. Construct an equilateral triangle inscribed in circle T shown below. [Leave all construction marks.]



27. To find the distance across a pond from point B to point C , a surveyor drew the diagram below. The measurements he made are indicated on his diagram.



Use the surveyor's information to determine and state the distance from point B to point C , to the *nearest* yard.

$$\frac{120}{x} = \frac{230}{315}$$

$$230x = 120(315)$$

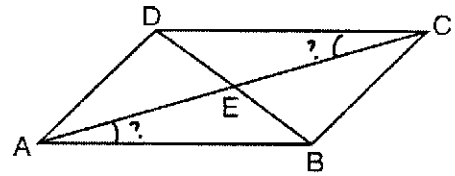
$$\frac{230x}{230} = \frac{37800}{230}$$

$$x = 164.3478261$$

$$x \approx 164 \text{ yds.}$$

28. In parallelogram $ABCD$ shown below, diagonals \overline{AC} and \overline{BD} intersect at E .

Prove: $\angle ACD \cong \angle CAB$

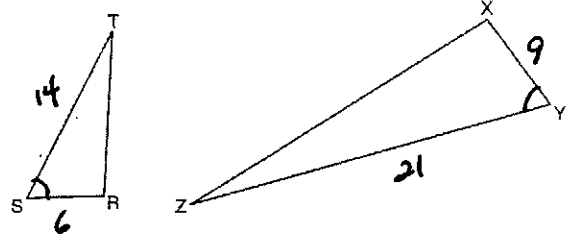


STATEMENTS	REASONS
① $\square ABCD$	① Given
② $\overline{AB} \parallel \overline{CD}$	② opp. sides of a \square are \parallel
③ $\angle ACD \cong \angle CAB$	③ if 2 \parallel lines are cut by a transv. then alternate interior \angle s are \cong

29. Triangles RST and XYZ are drawn below. If $RS = 6$, $ST = 14$, $XY = 9$, $YZ = 21$, and $\angle S \cong \angle Y$, is $\triangle RST$ similar to $\triangle XYZ$? Justify your answer.

$$\frac{9}{6} = \frac{21}{14}$$

$$126 = 126 \checkmark$$

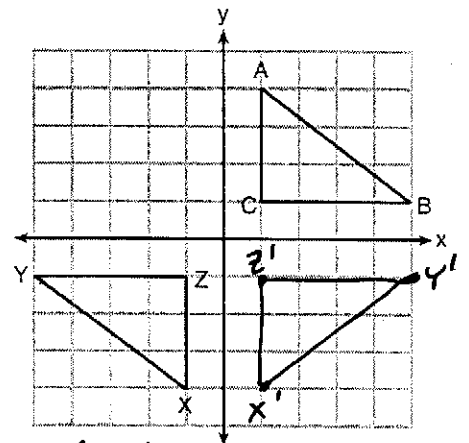


Yes, the Δ s are similar by SAS similarity

30. In the diagram below, $\triangle ABC$ and $\triangle XYZ$ are graphed.

Use the properties of rigid motions to explain why $\triangle ABC \cong \triangle XYZ$.

Two Δ s are \cong if there exists a sequence of rigid motions that maps one onto the other. Since $\triangle ABC$ is the image of $\triangle XYZ$ after a reflection over the y -axis followed by a reflection over the x -axis and reflections are rigid motions, the Δ s are \cong .



31. The endpoints of \overline{DEF} are $D(1,4)$ and $F(16,14)$. Determine and state the coordinates of point E , if $\overline{DE}:\overline{EF} = 2:3$.

$$\frac{2}{5}$$

$$x_1 + \frac{a}{b}(x_2 - x_1), y_1 + \frac{a}{b}(y_2 - y_1)$$

$$1 + \frac{2}{5}(16-1)$$

$$4 + \frac{2}{5}(14-4)$$

$$1 + \frac{2}{5}(15)$$

$$4 + \frac{2}{5}(10)$$

$$E(7,8)$$

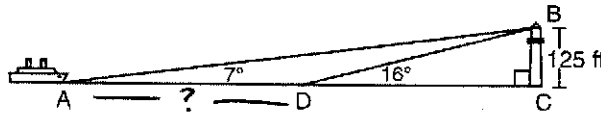
$$1+6$$

$$7$$

$$4+4$$

$$8$$

32. As shown in the diagram below, a ship is heading directly toward a lighthouse whose beacon is 125 feet above sea level. At the first sighting, point A , the angle of elevation from the ship to the light was 7° . A short time later, at point D , the angle of elevation was 16° .



To the nearest foot, determine and state how far the ship traveled from point A to point D .

$$\tan 16^\circ = \frac{125}{DC}$$

$$\tan 7^\circ = \frac{125}{AC}$$

$$DC = \frac{125}{\tan 16^\circ}$$

$$AC = \frac{125}{\tan 7^\circ}$$

$$DC = 435.9268055$$

$$AC = 1018.043303$$

$$AD = 1018.043303 - 435.9268055$$

$$AD = 582.1164975$$

$$AD \approx 582 \text{ ft}$$

33. Triangle ABC has vertices with $A(x,3)$, $B(-3,-1)$, and $C(-1,-4)$. Determine and state a value of x that would make triangle ABC a right triangle. Justify why $\triangle ABC$ is a right triangle. [The use of the set of axes below is optional.]

$$m_{\overline{BC}} = \frac{-4+1}{-1+3} = \frac{-3}{2}$$

$$A(3,3)$$

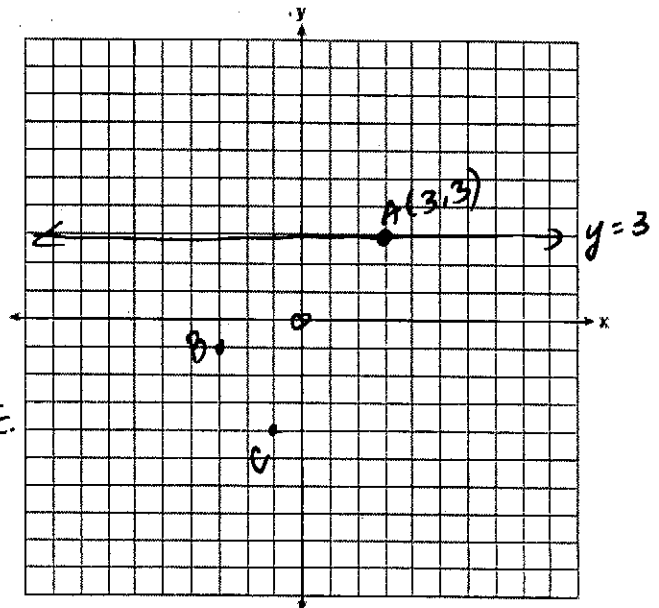
$$x=3$$

$$\perp m = \frac{2}{3}$$

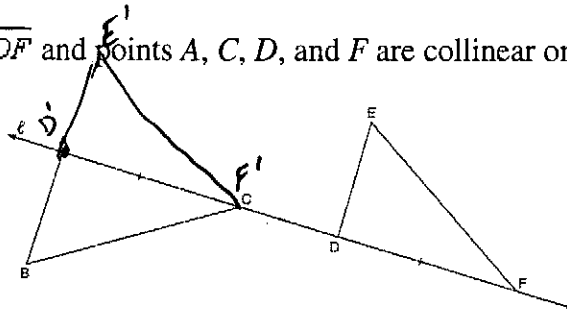
$$m_{\overline{AB}} = \frac{2}{3}$$

Since the slopes of \overline{AB} and \overline{BC} are negative reciprocals, $\overline{AB} \perp \overline{BC}$.

\perp lines form rt. \angle s so $\angle B$ is a rt \angle . $\triangle ABC$ with a rt \angle is a rt \triangle so $\triangle ABC$ is a rt \triangle .



34. In the diagram below, $\overline{AC} \cong \overline{DF}$ and points A, C, D, and F are collinear on line l .



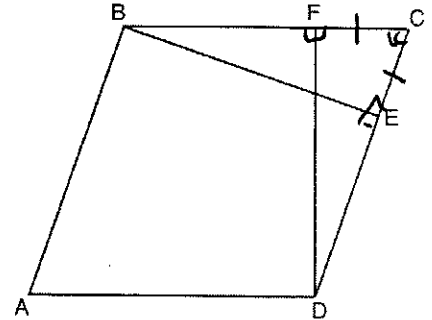
Let $\triangle D'E'F'$ be the image of $\triangle DEF$ after a translation along l , such that point D is mapped onto point A. Determine and state the location of F' . Explain your answer. Let $\triangle D''E''F''$ be the image of $\triangle D'E'F'$ after a reflection across line l . Suppose that E'' is located at B. Is $\triangle DEF$ congruent to $\triangle ABC$? Explain your answer.

F' would be translated to C. A translation is a rigid motion so distance is preserved. Since $\overline{AC} \cong \overline{DF}$, $\overline{AC} \cong \overline{D'F'}$.

$\triangle DEF$ is congruent to $\triangle ABC$ since $\triangle DEF$ maps onto $\triangle ABC$ after a sequence of rigid motions. Since a translation and reflection are rigid motions preserving distance & measure, the \triangle s are \cong .

35. In the diagram of parallelogram ABCD below, $\overline{BE} \perp \overline{CE}$, $\overline{DF} \perp \overline{BF}$, $\overline{CE} \cong \overline{CF}$.

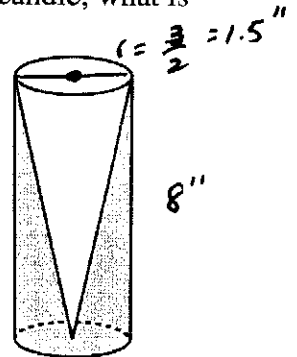
Prove ABCD is a rhombus.



STATEMENTS	REASONS
① $\square ABCD$, $\overline{BE} \perp \overline{CE}$, $\overline{DF} \perp \overline{BF}$, $\overline{CE} \cong \overline{CF}$	① Given
② $\angle CFD$ & $\angle CEB$ are rt. \angle s	② \perp lines form rt. \angle s
③ $\angle CFD \cong \angle CEB$	③ All rt. \angle s are \cong
④ $\angle C \cong \angle C$	④ Reflexive Prop.
⑤ $\triangle CFD \cong \triangle CEB$	⑤ ASA \cong ASA
⑥ $\overline{CD} \cong \overline{CB}$	⑥ CPCTC
⑦ ABCD is a rhombus	⑦ A \square w/ a pair of \cong consecutive sides is a rhombus

36. Walter wants to make 100 candles in the shape of a cone for his new candle business. The mold shown below will be used to make the candles. Each mold will have a height of 8 inches and a diameter of 3 inches. To the nearest cubic inch, what will be the total volume of 100 candles?

Walter goes to a hobby store to buy the wax for his candles. The wax costs \$0.10 per ounce. If the weight of the wax is 0.52 ounce per cubic inch, how much will it cost Walter to buy the wax for 100 candles? If Walter spent a total of \$37.83 for the molds and charges \$1.95 for each candle, what is Walter's profit after selling 100 candles?



$$V_{\text{cone}} = \frac{1}{3} \pi r^2 h$$

$$V_{\text{cone}} = \frac{1}{3} \pi (1.5^2)(8)$$

$$= 6\pi \text{ in}^3$$

$$V_{100} = 100(6\pi)$$

$$= 600\pi$$

$$= 1884.96$$

$$\approx \underline{1885 \text{ in}^3}$$

volume of
100 candles

$$600\pi \text{ in}^3 \cdot \frac{0.52 \text{ oz}}{1 \text{ in}^3} = 312\pi \text{ oz}$$

$$312\pi \text{ oz} \cdot \frac{\$0.10}{1 \text{ oz}} = \$98.0177$$

\$98.02 for 100 candles' wax

$$\begin{array}{r} 1.95 \\ \times 100 \\ \hline \$195 \text{ earned} \end{array}$$

$$195 - 98.02 - 37.83$$

$$= \underline{\$59.15 \text{ profit}}$$

