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WORKING PAPER N° 2023 – 13

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April 19, 2023

Preliminary

Abstract

Confronted with political opposition to the implementation of efficient carbon pricing, climate policy relies on alternative policy interventions, at a cost in terms of welfare and public finance. In order to evaluate this cost, this paper studies, in the context of the energy transition, second best climate policies constrained to keeping a constant level of the carbon tax and combining it with subsidies to carbon-free electricity generation. This subsidies can take the form of a feed-in premium paid to electricity produced from carbon-free sources, or of subsidies to investment in green capacity. Within a stylized dynamic model where energy may be produced with fossil or carbon-free sources and climate policy aims at satisfying a carbon budget, we define and characterize the carbon pricing gap. We show that if the constant carbon tax is small and therefore the carbon pricing gap large, the subsidy to carbon-free sources should be so large to foster rapid build up of green capacity that it would imply large investment costs and huge financial burden on the public budget, and a large welfare loss. We calibrate the model to the European energy market to obtain orders of magnitude of the effects.

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Key words: Energy transition; Carbon tax; Subsidies; FIP; Carbon-free energy; Policy acceptability.

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1 Introduction

To get rid of fossil fuels, the energy transition involves two crucial steps: decarbonising electricity production and electrifying industry, transport and building uses. The most direct and natural way to get rid of coal and gas in electricity generation is to make them more and more expensive, with a carbon price that increases over time to reflect our shrinking carbon budget. There is a wide consensus among economists to support carbon pricing, through a carbon tax or a cap-and-trade, as the best if not the only climate policy.[1] However, an overview of the direct pricing of carbon emissions around the globe makes clear that there is a substantial gap between the suggested and the observed price of carbon (OCDE, 2021), even though carbon pricing is getting more and more adopted (World Bank, 2022).

Would carbon pricing be popular among economists only? In several countries, the prospect of an increasing carbon tax has received strong political opposition, as exemplified by the famous Gilets Jaunes protest in France in 2018. Since then, the carbon tax has remained stuck at its 2018 level, and a resumption of its increase is completely absent from the public debate. In the United States, carbon pricing at the federal level seems out of reach, even though a few states (California, Oregon, Massachusetts) have implemented cap-and-trade schemes. The Inflation Reduction Act of August 2022 consists of a subsidy package of 391 billions of US dollars over ten years for energy and climate, among which 41% are production and investment tax credits for clean electricity (Committee for a Responsible Federal Budget,[2] 2022).[3] The European Union has also amply subsidized carbon-free energy in the past decades, and in 2020 renewable energy subsidies have reached 80 billions euros (European Commission, 2022), which is roughly twice as much as US annual subsidies. However, these subsidies come along with some carbon pricing through the Emissions Trading Scheme, targeting large energy-intensive companies. Though the carbon price on the EU-ETS has been very low in the past, it is not so any more and the European ambition is clearly increasing (see the Fit for 55 policy package)[4]. But fear of a European Gilets jaunes protest is clearly present: the current proposal (as for April 2023) is to implement in 2027 a second cap-and-trade scheme that will cover transport, building use and small businesses, and to cap the carbon price on this market at 45€/tCO_2, a level close to the current French carbon tax, at least until 2030. Hence, mitigation policies do not rely exclusively or even mainly on pricing carbon emissions. They typically include subsidies, which do not face the same political opposition (Klenert et al., 2018, Douenne and Fabre, 2022, Dechezleprtre et al., 2022).

We build in this paper a stylized model of the energy transition taking accounts of these facts.

[1] See for instance the 2021 statement on carbon pricing by the European Association of Environmental and Resource Economists at https://www.eaere.org/statement/

[2] https//www.crfb.org

[3] Local-content requirements are linked to these subsidies, which make the Inflation Reduction Act not only a climate policy but also an industrial policy, potentially protectionist. But this is another story.

[4] https://www.consilium.europa.eu/en/policies/green-deal/fit-for-55-the-eu-plan-for-a-green-transition/

Initially, energy consumption comes mostly from fossil sources. Making them progressively more expensive through carbon pricing would encourage investment in carbon-free electricity generation capacity (renewable or nuclear energy) to replace them. But as long as the demand for energy is greater than the carbon-free electricity supply, the residual demand is met by fossil fuels, and it is, according to the principle of marginal cost pricing, the price of the latter that sets the price of electricity. The price of electricity therefore increases with the carbon price throughout the transition. When the transition is achieved however, the price of electricity begins to fall, because carbon-free sources are very capital-intensive but have very low variable production costs. If the rise in the price of electricity during the transition is deemed socially unacceptable, society can very well choose a different route to achieve the transition: waive the carbon price or set it at a low level, and subsidize the carbon-free electricity production capacity (for instance through Feed-in Premiums) or the investment in this capacity (for instance through investment tax credits). This time the transition takes place without the price of electricity increasing and therefore without consumption decreasing.

We study the consequences of choosing the second route. In a decentralized set-up, the regulator is unable to implement the first best climate policy which would consist in an increasing carbon price. He is only able to charge a constant carbon tax, insufficiently high to ensure, on its own, that the carbon budget is respected. In a second best framework, we derive the additional subsidies that fill the policy gap while maximizing welfare. The amount of subsidies needed to achieve the decarbonisation objective is all the higher the greater the difference between the second best optimal carbon tax and the effective carbon tax. We name this difference the carbon pricing gap. The subsidies have to be financed, here by lump sum taxes, which puts a strain on public finances that is all the greater when the effective carbon tax is low. Moreover, to ensure a non-decreasing energy consumption during the transition while fighting against the excessive use of fossil fuels without increasing their price, the economy must invest too much and too quickly in carbon-free electricity production capacities, and, most of the time, the transition is delayed. To sum up, forgoing a rising carbon price in favor of subsidies to carbon-free electricity generation does allow the decarbonisation objective to be achieved, but (most of the time) later, at a cost to public finance, and at the cost of a welfare loss, the welfare cost of acceptability.

The framework is a simple version of Pommeret and Schubert (2022) and Pommeret et al. (2022). In these papers, carbon-free energy sources are not described as a backstop technology, abundant but more expensive than fossil fuels, as it is the case in older literature à la Hotelling (1931), but as energies which can only be harnessed if the appropriate dedicated capital infrastructures are built. The former paper focuses on the fact that renewable energies are non-dispatchable and are not continuously available. The latter considers the scarcity of raw materials for investing in green capital. In the present paper, we abstract from the intermittency issue and suppose that storage solutions exist (for instance batteries for the short run and hydrogen electrolysis and methanation for the longer run) at the cost of a more expensive electricity system. We also abstract from the material content of green capital.[5] Indeed, these two important dimensions of the energy transition are not central to the focus of this paper, which is to conduct an in-depth analysis of second best climate policy under a carbon pricing gap due to political economy constraints.

This paper contributes to the literature on the climate policy instrument design (reviews are provided by Goulder and Parry, 2008 and Phaneuf and Requate, 2017) by adding the acceptability dimension embedded in this choice. This literature has mostly analyzed the design of second best climate policy relying on instruments other than carbon pricing through the lenses of industrial organization applied to energy and environmental economics (e.g. Requate, 2015). In most cases, these analyses encompass additional market failures on top of the externality for greenhouse gas emissions,[6] and consider that the policymaker cannot freely manage as many instruments as there are market failures to be corrected.[7] In this vein, Fischer et al. (2021) establish the optimal adjustment of the available free tools to compensate for the constrained ones, and evaluate the implied welfare loss with respect to the unfeasible first best reference point. Second best climate policy is also analyzed using quantitative prospective modeling of the energy transition scenarios. For instance, Stock and Stuart (2021) use a partial equilibrium dynamic model of the electricity sector in the US and consider different combinations of the three main instruments of the bill proposed by the administration: clean energy standards, tax credits to investment in low carbon technologies, and a carbon tax. Moreover, the second best policy approach is at the heart of the double-dividend hypothesis, due to interactions between environmental regulation and preexisting distortions due to the fiscal system in static general equilibrium settings (e.g. Goulder et al., 1997). The general equilibrium approach has been extended to consider the endogenous dynamics resulting from accumulation or innovation, namely in integrated assessment models that quantitatively appraise the welfare cost of relying on subsidies to renewables rather than on carbon pricing. Kalhkul et al. (2013) show that the welfare costs of renewable energy subsidies are multiple times higher than first-best mitigation costs under a carbon price policy. This is mainly due to their inability to crowd out fossil fuels in a timely manner. Results further worsen in case of –even small– deviations from the second best optimum. In addition, Rezai and van der Ploeg (2017) stress that the welfare costs also significantly increase in case of lack of credibility of second best policies.[8] In as much we also

[5] Throughout the paper we use interchangeably the expressions clean, green, carbon-free and decarbonised energy and capital. On could however argue that they are not interchangeable because of the ambiguous status of nuclear energy. As we apply the model to the European energy transition where nuclear production capacities actually exist and new investments are planned for the near future, and as we do not want to enter an unproductive debate about what is clean/green and what is not, we retain this convention.

[6] Relevant for climate policies are those in the energy sector, aimed at coping with positive dynamic externalities from intentional investment to promote innovation or from learning-by-doing, or externalities from energy efficiency investment due to imperfect appropriation of the private returns or behavioral and information biases.

[7] Another typical issue is the one on overlapping regulations, as for instance for the interaction between national policies and the EU cap and trade CO_2 scheme (e.g. Böhringer and Rosendahl, 2010).

[8] Concerning renewable electricity quotas, Goulder et al (2016) compare the Clean Energy Standards to carbon cap and trade regulations using a static general equilibrium framework. Amigues et al (2022) consider the case of renewable portfolio standards in a dynamic general equilibrium setting with a carbon ceiling, comparing the floor to

rely on a dynamic general equilibrium analysis, these latter two articles are closest to our. However, we focus on a unique market failure (even though we introduce learning by doing in the empirical application) and we are able derive analytical results characterizing the trajectories.

The structure of the paper is the following. Section 2 presents the decentralized equilibrium of the economy. Section 3 analyzes the first best policies. The case of a carbon tax constrained to be constant is studied in section 4. Section 5 contains a numerical illustration of the results in the case of the European energy transition. Section 6 concludes.

2 A decentralized energy transition

We begin by presenting the main assumptions on preferences, technology and resource constraints, then characterize the behaviors and the decentralized equilibrium of the economy.

We consider a closed economy with a representative household consuming a generic good and electricity, and study it in continuous time. Electricity, $e(t)$, is produced with a linear technology either from the combustion of a flow of fossil resources, $x(t)$, or from the use of "green" capital, $Y(t)$, a stock of specialized equipment for carbon-free energy generation and storage:

$$e(t) = x(t) + \phi Y(t) \tag{1}$$

with $\phi > 0$ a measure of the efficiency of green electricity generation.

While the production of fossil resources entails no direct cost, their use implies carbon emissions, which accumulate in the atmosphere:

$$\dot{X}(t) = x(t) \tag{2}$$

where $X(t)$ denote cumulative carbon emissions and we normalize the emission coefficient to unity. Climate policy aims at limiting the temperature increase, compared to the pre-industrial equilibrium temperature. It is represented by a targeted carbon budget $\overline{X}$, i.e. the maximal amount of cumulative emissions compatible with the chosen temperature objective.

The stock of green capital depreciates at the constant rate $\delta \in (0, 1)$. It evolves with specific investment, $I(t)$, as follows:

$$\dot{Y}(t) = I(t) - \delta Y(t) \tag{3}$$

Investment entails capital adjustment costs, $C(I(t))$, assumed increasing and strictly convex (i.e. C', $C'' > 0$).

the share requirement regulatory approach.

Denoting by Y_0 the initial endowment in green capital, we can write the set of constraints that apply to the flow and stock variables as:

$$X(t) \leq \overline{X}, \quad x(t) \geq 0, \quad X(0) = 0 \text{ and } Y(0) = Y_0 \geq 0 \text{ given} \tag{4}$$

Four policy tools are available to the regulator: a carbon tax $\tau(t)$; a feed-in premium (FIP) for electricity from carbon-free sources $\sigma(t)$; a subsidy to investment in green capital $s(t)$; a tax on electricity consumption $\theta(t)$. The public budget is balanced through lump-sum transfers or taxes, $\mathcal{T}(t)$, to the household:

$$\tau(t)x(t) + \theta(t)e(t) = \sigma(t)\phi Y(t) + s(t)I(t) + \mathcal{T}(t) \tag{5}$$

The regulator chooses the policy instruments, households demand electricity, power companies supply electricity, and to do so they invest in green capital and demand fossil resource inputs, and fossil resource producers supply the latter. Here we characterize the behaviors of these agents for given policy tools, and then present the main features of the general dynamic equilibrium.

2.1 Households

The representative household maximizes his intertemporal utility. Utility is derived from the consumption of a generic good, z, and electricity, e. The representative household is characterized by an instantaneous utility function that is assumed to be quasi-linear in the generic good, taken as numeraire, and concave in electricity consumption, according to $u(e(t))$, with $u' > 0$ and $u'' < 0$. He applies a constant discount rate, $\rho > 0$. His behavior on the electricity market is characterized by the demand function:[9]

$$e(t) = u'^{-1}(p_e(t) + \theta(t)) = e(p_e(t) + \theta(t)) \tag{6}$$

Also, it implies the equality of the interest and discount rates: $r(t) = \rho \ \forall t$.

2.2 Power utilities

Perfectly competitive utilities produce electricity from either fossil resources or from a specific green capital. Investment in green capital is irreversible. The representative power utility acts under perfect competition on the electricity market and on the markets for natural resources, where it takes prices as given. The firm seeks to maximize the present value of its profits, taking into account that building up its green capital to produce carbon-free energy implies adjustment costs.

[9] See Appendix A for the derivations of the optimality conditions of this section.

It therefore solves an intertemporal program, based on the expected evolution of the electricity price p_e, the prices of the fossil resource p_x, as well as policy tools, namely, the carbon tax $\tau(t)$, the FIP $\sigma(t)$ and the subsidy $s(t)$.

When some of the electricity is optimally produced using fossil inputs, the seller price of electricity equals the cost of fossil resource use, comprehensive of the price of the resource and of the carbon tax:

$$x(t) > 0 \quad \Leftrightarrow \quad p_e(t) = p_x(t) + \tau(t) \tag{7}$$

On the other hand, when the price of electricity is smaller than the cost of fossil inputs, fossil inputs are not used and electricity is produced with carbon-free sources only:

$$x(t) = 0 \quad \Leftrightarrow \quad p_e(t) < p_x(t) + \tau(t) \tag{8}$$

Power utilities use fossil inputs only to cope with demand above the base power $\phi Y(t)$ produced by the already installed capacity with nil marginal cost. In other words, they employ fossil resources to satisfy residual demand: $x(t) = e(t) - \phi Y(t)$ is positive if the demand $e(t)$ is above the base load $\phi Y(t)$.

Denoting μ_d the private value of green capital, costate of Y in the representative firm's programme, the first order conditions characterizing the accumulation of green capital by electricity producers are, when investment is strictly positive:

$$C'(I(t)) = \mu_d(t) + s(t) \tag{9}$$

$$\dot{\mu}_d(t) = (r(t) + \delta)\mu_d(t) - \phi(p_e(t) + \sigma(t)) \tag{10}$$

When $\mu_d(t) < C'(0)$ there is no investment in green capital. When $\mu_d(t) > C'(0)$, according to equation (9) investment in green capital takes place up to the point where the marginal investment cost is equal to the private value of green capital, augmented by the subsidy to investment. This condition implicitly determines the optimal investment level, $I(\mu_d(t) + s(t))$. Equation (10) shows how the private value of green capital evolves over time: it is driven by the electricity price augmented by the FIP.

2.3 Fossil resources market

Our focus being on an effective climate policy, which makes the environmental problem more stringent than resource scarcity, we ignore the scarcity of fossil resources. Producers act as if their resources were abundant, so that, rather than solving an intertemporal optimization problem, they maximize their current profit at each date. Given that the production costs are assumed nil and that the market is perfectly competitive, the equilibrium seller's price for fossil resources is nil:

$$p_x(t) = 0 \tag{11}$$

2.4 Equilibrium

In equilibrium, when fossil resources are used to produce power, (10) can be written using the equality of the interest and discount rates, and equations (7) and (11), to get

$$\dot{\mu}_d(t) = (\rho + \delta)\mu_d(t) - \phi(\tau(t) + \sigma(t)) \quad \text{if } x(t) > 0 \tag{12}$$

Moreover, the equilibrium on the electricity market is obtained by combining (6), (7) and (11), to get

$$x(t) = e\left(\tau(t) + \theta(t)\right) - \phi Y(t) \quad \text{if } x(t) > 0 \tag{13}$$

Notice that if $Y_0 > \frac{1}{\phi} e\left(\tau(0) + \theta(0)\right)$, that is in the case of a large initial green capital stock and high carbon and electricity taxes, fossil is never used. We suppose in what follows that we are not in this case, because, in particular, the initial green capital is small. Then, according to (13), we can define the date T when the fossil inputs stop being used as:

$$Y(T) = \frac{1}{\phi} e\left(\tau(T) + \theta(T)\right) \tag{14}$$

To avoid an unnecessary multiplication of cases we also suppose that the initial green capital is small enough for investment to take place initially: $I(0) > 0$, i.e. $C'(0) < \mu_d(0)$.

After T, power relies exclusively on carbon-free sources, i.e. $x(t) = 0$, and (10) can be written using (6), to get

$$\dot{\mu}_d(t) = (\rho + \delta)\mu_d(t) - \phi[u'(\phi Y(t)) - \theta(t) + \sigma(t)] \quad \text{if } x(t) = 0 \tag{15}$$

In this case the supply on the electricity market is perfectly rigid, so that the full incidence of the tax on consumption is on the power utility. As a consequence, it is conceivable that the level of the consumption tax be so high that some capacity could remain idle

$$e(t) = e(\theta(t)) \leq \phi Y(t) \tag{16}$$

Of course, there is no reason to impose such a high level of taxation.

Let us notice that when fossil resources are not exploited, the consumption tax and the FIP are equivalent policy instruments, since they only affect the evolution of the private value of green capital, and do so in the same way (see (15)).

2.5 Regulator

The objective of the regulator is to design the climate policy tools in order to maximize social welfare, taking into account the climate constraint and the reaction functions of households. His objective function reads:

$$W = \int_0^{\infty} e^{-\rho t} \left[u(x(t) + \phi Y(t)) - C(I(t)) \right] dt \tag{17}$$

3 First best policies

We first characterize the optimal energy transition. [10] Second, we recall that optimal carbon pricing is enough to set the decentralized equilibrium of the economy on the optimal path. We then show that, absent the carbon tax, combining the FIP with the tax on electricity consumption still allows to bring the economy on the optimal path.

3.1 Optimal energy transition

The benevolent social planner seeks to maximize society's net surplus W defined in equation (17), subject to the technology and resource constraints (1)-(3), as well as the non-negativity and carbon budget constraints, for given initial conditions (4).

The optimal energy transition is characterized in the following proposition, where $\lambda(t)$, the carbon value, and $\mu(t)$, the value of the green capital, are the costate variables respectively associated to cumulative emissions $X(t)$ and green capital $Y(t)$.

PROPOSITION 1. Optimal energy transition. *For a small green capital endowment, the optimal path is determined by the vector $\{\mu^{\circ}(0), \lambda^{\circ}(0), T^{\circ}\}$, such that the economy undergoes two phases: a carbon era when $x^{\circ}(t) > 0$ $\forall t < T^{\circ}$, followed by a clean era when $x^{\circ}(t) = 0$ for $t \geq T^{\circ}$, characterized by the system of differential equations:*

$$\dot{Y}^{\circ}(t) = I\left(\mu^{\circ}(t)\right) - \delta Y^{\circ}(t) \quad \forall t \geq 0 \tag{18}$$

$$\dot{\lambda}^{\circ}(t) = \rho \lambda^{\circ}(t) \; \forall t < T^{\circ} \tag{19}$$

$$\dot{\mu}^{\circ}(t) = \begin{cases} (\rho+\delta)\mu^{\circ}(t) - \phi\lambda^{\circ}(t) & \forall t < T^{\circ} \\ (\rho+\delta)\mu^{\circ}(t) - \phi u'(\phi Y^{\circ}(t)) & \forall t \geq T^{\circ} \end{cases} \tag{20}$$

[10] In essence, the analysis in this section is a special case of Pommeret and Schubert (2022) and of Pommeret et al. (2022).

Electricity consumption is driven by the carbon value in the carbon era, and by the level of green capital in the clean era:

$$e^\circ(t) = \begin{cases} x^\circ(t) + \phi Y^\circ(t) = e(\lambda^\circ(t)) & \forall t < T^\circ \\ \phi Y^\circ(t) & \forall t \geq T^\circ \end{cases} \tag{21}$$

The carbon budget $\overline{X}$ *is exhausted at date* T°*, satisfying*

$$\lambda^\circ(T^\circ) = u'\left(\phi Y^\circ\left(T^\circ\right)\right) \tag{22}$$

The optimal path reaches asymptotically the steady state (Y^*, μ^*)*, defined by:*

$$\begin{cases} \frac{\rho+\delta}{\phi} C'(\delta Y^*) = u'(\phi Y^*) \\ \mu^* = C'(\delta Y^*) \end{cases} \tag{23}$$

which is unique and saddle-path stable.

The proof is relegated to Appendix B.

In the carbon era, the carbon value follows the Hotelling rule. It drives electricity consumption and the evolution of the value of green capital; in turn, the value of green capital drives its accumulation. Along the optimal trajectory, electricity consumption is non-monotonous. At first, it is decreasing to reflect the ever increasing stringency of the carbon budget. When the later is exhausted however, consumption increases along with production capacity from carbon-free energy sources.

3.2 First best carbon pricing

Since there is one market imperfection related to the carbon budget, one policy instrument is sufficient to set the decentralized equilibrium of the economy on the optimal path.

Proposition 2. Optimal carbon pricing. *The decentralized equilibrium coincides with the optimal path when the carbon tax equals the carbon value along the optimal energy transition:*

$$\tau^{fb}(t) = \lambda^\circ(t), \ t \leq T^\circ \tag{24}$$

where λ° *and* T° *are defined in Proposition 1, and no other instrument is used:* $\sigma^{fb}(t) = s^{fb}(t) = \theta^{fb}(t) = 0$.

The optimal policy calls for an increasing carbon tax. As argued in the introduction, in practice such a policy raises political opposition, making its implementation perilous.

3.3 Combining the FIP with the tax on electricity consumption

Remember that during the carbon era the optimal consumption path is decreasing, although the production of electricity from carbon-free sources is increasing. This is possible only if the use of fossil resources declines. Absent carbon pricing, the private marginal cost of using fossil inputs for the power utility is nil during this phase. As a result, there are no incentives to reduce production of electricity from fossil resources.

Nevertheless, there is one possibility to induce the power utility to use ever less fossil inputs: as these inputs are used to satisfy residual demand, the regulator can induce a decline in their use by appropriately reducing the demand for electricity, thanks to the consumption tax.

In fact, setting $\tau(t) = 0$ in (13), we see that at the decentralized equilibrium $u'(x(t) + \phi Y(t)) = \theta(t)$ while at the optimum $u'(x(t) + \phi Y(t)) = \lambda^\circ(t)$. Hence, along a trajectory where green capital investment is optimal, the fossil input use could be reduced at the optimal pace by setting a consumption tax equal to the carbon value:

$$\widetilde{\theta}(t) = \lambda^\circ(t) \tag{25}$$

However, there is no reason for investment in green capital to follow the optimal path when only the electricity consumption tax is implemented. Inspecting the evolution of the green capital value in (12) when $\tau(t) = 0$, and comparing with its optimal path in (20), one sees that the consumption tax does not influence the value of green capital, and cannot therefore cope with the inability of the constant carbon tax to signal the increasing social value of carbon. However, the FIP could be increased in order to solve the problem. Its optimal value would indeed be identical to the electricity consumption tax. Hence, subsidies would be entirely financed through the tax on electricity consumption.

The tax on electricity consumption is used to make the electricity demand decrease to prevent the use of fossils, whereas the FIP is used to undo the consequences of the former on carbon-free capacity accumulation.

To summarize:

PROPOSITION 3. Optimal energy transition (2). *The optimal trajectory can be implemented in a decentralized equilibrium with no carbon tax by levying a tax on electricity consumption* $\widetilde{\theta}(t)$ *defined by (25), while subsidizing electricity production from carbon-free sources through a FIP* $\widetilde{\sigma}(t) = \widetilde{\theta}(t)$. *This policy implies a surplus of the public budget* $\mathcal{T}(t) = \lambda^\circ(t)x^\circ(t)$ *during the carbon era and a balanced budget during the clean era.*

A remark is worthwhile: if potential political opposition to an increasing path of the carbon tax is the underlying constraint on policy design, one suspects that there would be political opposition also to an increasing path of taxes on electricity consumption as in (25). Proposition 3 may therefore be of little practical guidance.

We now turn to the question of what can be done when the regulator can only commit to a constant carbon tax.[11]

4 Second best policy with a constant carbon tax

For the remainder of the article, we assume that the carbon tax is set at a constant level $\widetilde{\tau}$ for the sake of acceptability and we abandon the tax on electricity consumption. We consider that the carbon tax is complemented first by a FIP, and second by a subsidy to investment in green capacity.

4.1 Relying on a FIP

Consumers, electricity producers and the regulator play a Stackelberg policy game, where the leader is the regulator (see Dockner at al, 2000, chapter 5). At date 0, the constant carbon tax $\widetilde{\tau}$ is exogenously set at a level acceptable by society and the regulator, as a first mover, announces the FIP path $\sigma(t)$, and commits to this plan. Taking into account this information on climate policy, households optimally decide how much electricity to consume, and electricity producers choose the electric mix and the amount of green capital to install. The game is solved by backward induction: the regulator, knowing the agents' best responses to his policy, proceeds to choose the FIP path maximizing his objective function while complying with the carbon budget, conditional to the carbon tax $\widetilde{\tau}$.

Let us describe first the clean era. As there is no externality in this phase, the private and social second best values of green capital are aligned and there is no reason to subsidize clean electricity production: $\sigma(t) = 0$.[12] Lemma 1 follows.

Lemma 1. The clean era at second best.

(i) The clean era is described by the same system of differential equations (18) and (20) as at the first best. Green capital and its shadow value converge asymptotically along the saddle path to their steady state levels defined by (23). However, the clean era sharpts at a different date than at the first best, from a different level of green capital.

(ii) The initial level of green capital in the clean era is $Y^\sharp(\widetilde{T}^\sharp) = e(\widetilde{\tau})/\phi$, a decreasing function of the carbon tax. It is larger than at the first best iff $\widetilde{\tau} < \tau^{fb}(T^\circ) = \lambda^\circ(0)e^{\rho T^\circ}$.

[11] The analysis is essentially the same if the carbon tax to which the regulator commits is not constant but increasing over time at a rate lower than the discount rate. A justification could be that what is "acceptable" evolves over time.

[12] Appendix F presents an extension of the model where learning by doing causes the decrease of the cost of investment in green capital, and thereby introduces a motive for subsidizing green capital also in the clean era.

(iii) For a small enough carbon tax, $\widetilde{\tau} < \tau^{ov} \equiv u'(\phi Y^*)$, the initial level of green capital in the clean era is larger than its long run value, $Y^\sharp(\widetilde{T}^\sharp) > Y^*$: second best climate policy entails overshooting the long term accumulation target, to compensate for the weakness of the carbon tax.

See Appendix C for the proofs.

Notice that since $\widetilde{\tau}$ has been set to make carbon taxation acceptable, it will most plausibly be smaller than the optimal carbon value at the date of fossil phase out, $\tau^{fb}(T^\circ) = \lambda^\circ(T^\circ)$. Therefore the green capital accumulated at the onset of the clean era will be most plausibly larger at the second best than at the first best. However, at this stage, nothing can be said concerning the date of the switch to the clean era: a priori, the energy transition can be delayed or accelerated at the second best, compared to what is optimal.

We now characterize the second best climate policy in the carbon era. According to equations (6), (9) and (10) with $p_e = \widetilde{\tau}$ and $\theta = s = 0$, consumers and electricity producers' best responses to the carbon tax and the FIP are respectively:

$$e(t) = e(\widetilde{\tau})$$

and

$$\begin{cases} \dot{Y}(t) = I\left(\mu_d(t)\right) - \delta Y(t) \\ \dot{\mu}_d(t) = (\rho + \delta)\mu_d(t) - \phi\left(e(\widetilde{\tau}) + \sigma(t)\right) \end{cases}$$

We denote by $\zeta_1(t)$ the second best carbon value, shadow price of cumulative carbon emissions in the regulator's program, and by $\zeta_2(t)$ the second best social value of green capital, shadow value of the green capital stock. The regulator has to take into account the evolution over time of the private value of green capital, $\mu_d(t)$, which determines the investment effort. Therefore, $\mu_d(t)$ is treated in the regulator's program as a state variable, to which we associate a costate $\zeta_3(t)$. This costate can be interpreted as a measure of the extent to which the second best private and social values of green capital differ. We then prove in Appendix C the following Proposition.

Proposition 4. The carbon era at second best.

(i) The optimal trajectory cannot be implemented in a decentralized equilibrium with only a constant carbon tax and a FIP, as the electricity consumption is then constant up to the date of fossil phase out.

(ii) The optimal date of the switch to the clean era $\widetilde{T}^\sharp$ is such that the marginal benefit of delaying the switch is nil, which implies the continuity of the private value of green capital μ_d at $\widetilde{T}^\sharp$.

(iii) At each level of the constant carbon tax corresponds a second best carbon value, $\zeta_1^\sharp$, increasing over time at the discount rate, and which initial value is a decreasing function of the carbon tax. The

carbon pricing gap is the difference between this second best carbon value and the effective constant carbon tax, $\zeta_1^\sharp(t) - \widetilde{\tau}$*. It is therefore increasing over time.*

(iv) Given that $Y_0 < Y^*$*,*

a. *If the carbon tax is above* $\overline{\overline{\tau}} = u'(\phi Y_0)$*, the fossil resource is not used, and the optimal FIP is nil.*[13]

b. *If* $\widetilde{\tau} \in (\overline{\tau}, \overline{\overline{\tau}}]$*, the fossil resource is used, but the carbon pricing gap is always negative and therefore the FIP nil.* $\overline{\tau}$ *is defined by* $\overline{\tau} = \zeta_1^\sharp(\widetilde{T}^\sharp)|_{\overline{\tau}}$*.*[14]

c. *If* $\widetilde{\tau} \in (\widehat{\tau}, \overline{\tau}]$*, the carbon pricing gap is initially negative and the FIP nil, up to the date* T_0 *when the carbon pricing gap becomes nil. After* T_0 *the FIP compensates for the carbon pricing gap:*

$$\sigma^\sharp(t) = \zeta_1^\sharp(t) - \widetilde{\tau} \tag{26}$$

It is therefore increasing over time until $\widetilde{T}^\sharp$*, and optimally jumps down at* $\widetilde{T}^\sharp$ *from a positive value to zero.* $\widehat{\tau}$ *is defined by* $\widehat{\tau} = \zeta_1^\sharp(0)|_{\widehat{\tau}}$*.*

d. *If* $\widetilde{\tau} \in (\underline{\tau}, \widehat{\tau}]$*, the FIP is optimally provided from the sharpt, according to (26).* $\underline{\tau}$ *is the smallest constant carbon tax allowing the economy to satisfy the carbon budget constraint.*[15]

e. *If* $\widetilde{\tau} > \tau^{ov} = u'(\phi Y^*)$ *defined in Lemma 1, the accumulation of green capital is monotonically increasing toward* Y^**; if now* $\widetilde{\tau} < \tau^{ov}$*, there is overshooting of green capital, i.e.* $Y^\sharp(\widetilde{T}^\sharp) > Y^*$*.* τ^{ov} *can be larger or smaller than* $\widehat{\tau}$*.*

f. *If* $\widetilde{\tau} \leq \underline{\tau}^{ov}$*, the FIP is capped at a value* σ_{max} *to prevent an overshooting so large that it would entail disinvestment in the clean era.* $\underline{\tau}^{ov}$ *can be larger or smaller than* $\underline{\tau}$*; if it is smaller it is not relevant.*

(v) As the private value of green capital μ_d *is non-controllable, the second best policy is time-consistent.*

Result (ii) means that at the date chosen by the regulator for achieving fossil phase out, the electricity producer does not have any incentive for modifying the green capital accumulation path. Indeed, he chooses the accumulation path knowing in advance the evolution of the electricity price and of policy tools. His choice is optimal only if, at any date, he does not regret to own the chosen

[13] Actually taxing the production of electricity from carbon-free sources to induce consumption of some fossil resource until the carbon budget is met would be welfare improving (recall that there is no cost associated to fossil resource if the carbon budget is not exhausted).

[14] The notation means that $\overline{\tau}$ is the fixed point of the equation, with both terms on the right-hand-side depending on $\widetilde{\tau}$.

[15] We do not elaborate on this point because in the presence of fossil extraction costs it is possible to satisfy the carbon budget constraint with a nil carbon tax. The case $\underline{\tau} > 0$ is an uninteresting artefact coming from our simplifying assumption of no extraction costs.

amount of green capital. Since the stock of green capital is a continuous variable, it follows that the private value of green capital shall not change discontinuously. In fact, a sudden downward jump in the private value of green capital would imply a capital loss, that could be avoided by appropriately modifying the accumulation path.

Results (iii)–(iv) describe the different cases that may occur, depending on the level of the carbon tax $\widetilde{\tau}$. They are represented on Figure 1, which depicts the initial second best carbon value $\zeta_1(0)$ as a function of the carbon tax $\widetilde{\tau}$, in the case $\tau^{ov} < \widehat{\tau}$.

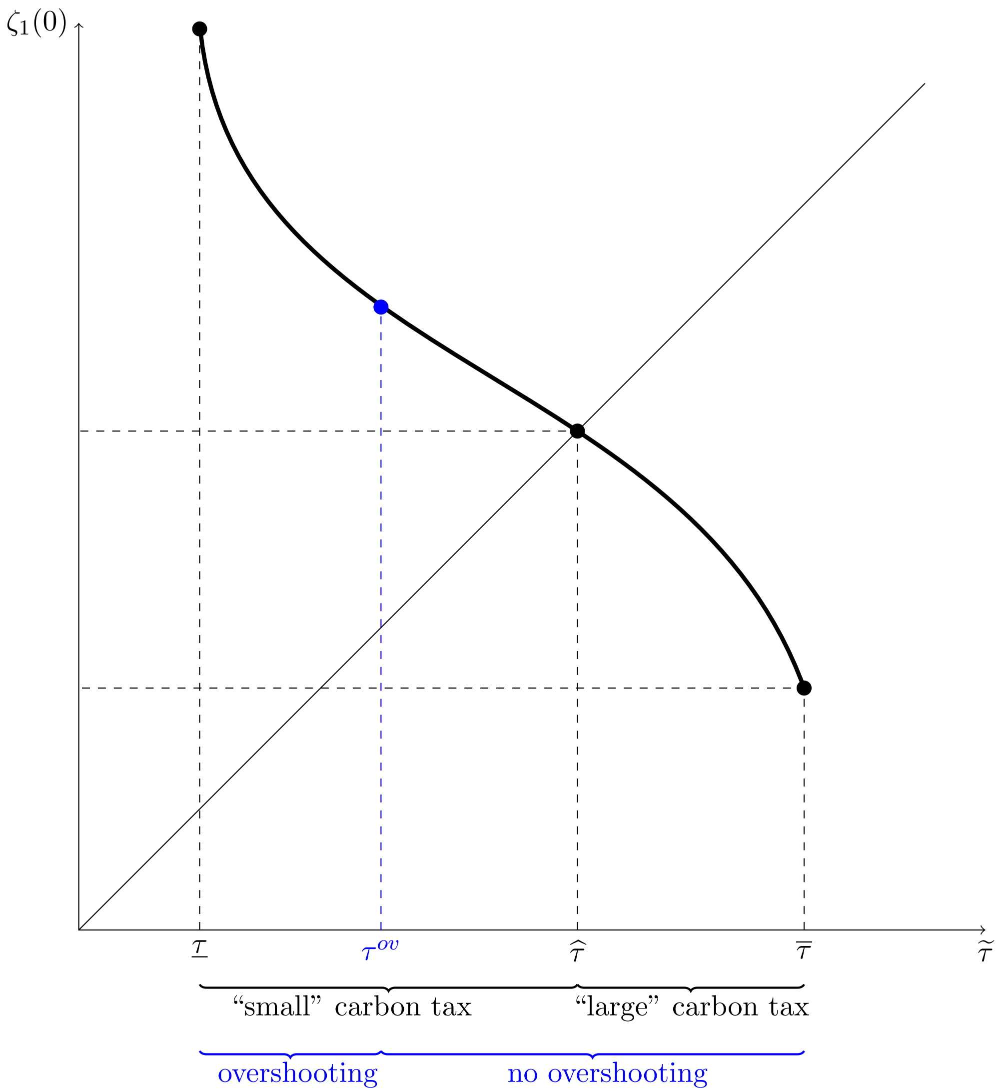


Figure 1: Initial second best carbon value as a function of the carbon tax. Case $\tau^{ov} < \widehat{\tau}$

Cases (iv) b corresponds to a very large constant carbon tax, large enough to trigger the phasing out of fossil fuels without any additional FIP. This case is clearly not plausible, as is Case (iv) a where fossil fuels are never used. Case (iv) c corresponds to a "large" carbon tax, larger than the initial second best carbon value, but becoming smaller than the second best carbon value (which increases at the discount rate) at a date T_0 predating fossil phase out. No additional FIP is provided before T_0, when the carbon pricing gap is negative and the private value of green capital higher than its social value. After T_0 the carbon pricing gap becomes positive, justifying the introduction of the FIP, which makes possible the equalization of the private and social values of green capital. Case (iv) d is the case of a "small" carbon tax, for which it is necessary to provide the FIP all along the carbon era. Finally, Case (iv) e defines the constant carbon tax τ^{ov} above which green capital monotonously increases along the carbon and clean eras and under which green capital overshoots its long run value in the carbon era. As τ^{ov} and $\widehat{\tau}$ are defined independently, τ^{ov} depending on the steady state value of green capital, itself a function of the preference and cost parameters, and $\widehat{\tau}$ depending on the stringency of the carbon budget constraint, their ranking is ambiguous. Everything else equal, a more stringent climate policy makes the second best carbon value increase, and increases the probability that $\tau^{ov} < \widehat{\tau}$.

Result (v) states an important property of the second best policy, its time-consistency. Indeed, open-loop Stackelberg equilibria give rise most of the time to time-inconsistent policy rules: the regulator has the incentive to change the policy if he recomputes it at a later time. Time-inconsistency would occur here if the regulator was able to manipulate the private value of green capital with his choice of the FIP. We show that this is not the case.

Notice that according to equation (26) $\sigma^\sharp(t)$ is increasing over time, as is $\zeta_1^\sharp(t)$, until the date of fossil phase out $\widetilde{T}^\sharp$. Just after $\widetilde{T}^\sharp$ it becomes nil. If the constant carbon tax is small the discrepancy between the revenues it provides and the cost of the FIP for public finance is large and increasing, which means that the regulator has to levy large and increasing lump-sum taxes on households. The implicit assumption here is that this does not pose any acceptability problem. It may be justified in several ways: one can argue that general equilibrium effects are badly understood, or, in the case they are actually understood, that subsidies are preferred to carbon pricing because the cost is more diffusely distributed in the population, or borne by different agents.

Besides the cost of the second best policy in terms of public finance, it has a cost in terms of welfare, that we name the welfare cost of acceptability. Giving an analytical expression of this welfare cost is out of reach. In the application of the model to the European energy transition (section 5) we compute numerically the value functions for different values of the constant carbon tax and compare them in order to evaluate this welfare cost of acceptability.

4.2 Imposing a balanced public budget

Studying second best policies is mainly driven by the objective to reduce the opposition to climate policies. According to the literature, this opposition is weaker when the revenues from the tax are used to finance subsidies to carbon-free energy production (Douenne and Fabre, 2022, Klenert et al., 2018). Therefore, we study the case where the regulator pays the FIP with the tax revenues, hence ensuring a balanced budget without transfers ($\mathcal{T}(t) = 0$):

$$\widetilde{\tau} x(t) = \sigma(t)\phi Y(t) \tag{27}$$

Substituting for the decentralized use of fossils (13) for $\theta = 0$, in the budget balance above, we see that the the FIP is at each date determined by the carbon tax and the stock of green capital as:

$$\sigma^c(t) = \widetilde{\tau}\left(\frac{e(\widetilde{\tau})}{\phi Y^c(t)} - 1\right) > 0 \ \forall t \leq \widetilde{T}^c \tag{28}$$

It decreases over time and then is nil from the date of fossil phase out, $\widetilde{T}^c$, onward.

First, imposing a balanced budget amounts to an additional constraint, necessarily generating an adverse effect on welfare. Second, it cannot ensure reaching both the climate and acceptability objectives. In case $\widetilde{\tau}$ is set exogenously to ensure acceptability, the FIP being out of the regulator's control there is no reason why the carbon budget should be met. There is only one level of the tax for which the carbon budget is met (see Appendix D). Nothing ensures that this carbon tax is lower than the initial optimal tax, hence potentially missing the acceptability target.[16] As a result, limiting the FIP to be exclusively financed out of carbon tax revenues does not seem to be a wise policy option.

4.3 Relying on a subsidy to investment in green capital

We now study the case where the regulator puts in place a subsidy to investment in green capital instead of a FIP to complement the constant carbon tax. The policy game is similar to the one implemented to design the optimal FIP. Given a constant carbon tax $\widetilde{\tau}$, the regulator announces and commits to a subsidy path $s(t)$. The subsidy is chosen by the regulator who takes the agents' best responses into account when maximizing his objective function under the carbon budget constraint. For the sake of brevity we only study the interior case where the carbon pricing gap is positive all along the carbon era ("small" carbon taxes).

Proposition 5. Constant carbon tax and subsidy to green investment.

(i) When the carbon tax is small enough for the carbon pricing gap to be positive from the sharpt, the subsidy is positive and fills the gap between the social and the private values of green capacity:

$$s^\sharp(t) = \zeta_2^\sharp(t) - \mu_d(t) \tag{29}$$

Equivalently, it is equal to the discounted present value of the future carbon policy gaps until the end

[16]See Pommeret et al. (2022) that exhibits numerically this case.

of the carbon era:

$$s^\sharp(t) = \int_t^{\widetilde{T}^\sharp} e^{-(\rho+\delta)(u-t)}(\zeta_1^\sharp(u) - \widetilde{\tau})du \tag{30}$$

The subsidy is continuous (and equal to zero) at the date of fossil phase out.

(ii) For a given carbon tax, there exists a unique path of subsidy to investment in green capital $s^\sharp$, leading to the same electricity consumption, the same timing of fossil phase out and the same accumulation of green capital as a FIP $\sigma(t)$ provided to electricity producers as described in Proposition 4. The relationship linking the two instruments is:

$$\dot{s}^\sharp(t) = (\rho + \delta)s^\sharp(t) - \phi\sigma^\sharp(t) \tag{31}$$

(iii) Choosing to fill the carbon pricing gap with a subsidy to investment in green capital rather than with a FIP induces a transfer from the public budget to the electricity producers, immaterial to households:

$$\int_0^{\widetilde{T}^\sharp} e^{-\rho t}s^\sharp(t)I^\sharp(t)dt = \int_0^{\widetilde{T}^\sharp} e^{-\rho t}\sigma^\sharp(t)\phi Y^\sharp(t)dt - s^\sharp(0)Y_0 \tag{32}$$

Proof. The proof is relegated to Appendix E. □

The main result in Proposition 5 hinges on the equivalence between using subsidies to investment or FIPs, which underlies one major distinguishing feature between the American and the European policy designs. The former relies mostly on tax credits for investments to expand green capacity,[17] while the latter relies more on feed-in premiums and similar subsidies to purchases of electricity from carbon-free production units.

5 Application to the European energy transition

In order to gain insights on the magnitude of the carbon pricing gap, the cost for public finance and the welfare cost associated to various choices of the constant carbon tax, we calibrate the model to the case of the European energy transition.

5.1 Extension of the model

We extend the model to include two features that do not add much from a theoretical point of view but are crucial from the empirical one: extraction costs of the fossil resources and learning by doing in carbon-free electricity generation. The existence of the learning by doing externality is well documented empirically. Rubin et al. (2015) provide a review of learning rates for electricity supply

[17] See the provisions of the 2022 Inflation Reduction Act.

technologies. These ranges vary widely across technologies and studies. The highest reported rate is of 25% per year for onshore wind. Way et al. (2022) report that “for several decades the costs of solar photovoltaics, wind, and batteries has dropped (roughly) exponentially at a rate near 10% per year”. Bollinger and Gilllingham (2019) attribute this drop to learning by doing, both “internal”, that is through cumulative experience, and “external”, that is through spillovers.

We make the assumption that fossils' extraction costs are linear, with a constant unit cost c_x. For carbon-free energies, we replace the investment cost function $C(I)$ by $C(I,Y)$, with $C_I > 0$, $C_Y < 0$, $C_{II} > 0$, $C_{YY} > 0$ and $C_{IY} < 0$, to take into account the learning by doing effect. We use to illustrate the following quadratic form:

$$C(I,Y) = c_1(Y)I + \frac{c_2}{2}I^2, \quad \text{with } c_1'(Y) < 0 \text{ and } c''(Y) < 0$$

The changes to the previous model implied by these extensions are presented in Appendix F.

The implementation of the first best requires to put in place in the carbon era a carbon tax equal to the social carbon value, $\tau^{fb}(t) = \lambda^\circ(t)$, and in both eras a subsidy to green capital proportional to the difference between the social value of green capital and its current unit cost, the coefficient of proportionality depending on the strength of the learning externality $-c_1'(Y)$:

$$\Sigma(t) = -C_Y = -\frac{c_1'(Y(t))}{c_2}(\mu_d(t) - c_1(Y(t)) \tag{33}$$

Notice that the regulator must provide the subsidy to green capital forever, to incentivize electricity producers to keep the green capital stock at the right level at the steady state, despite depreciation.

At the second best, the optimal subsidy to carbon-free electricity production is the sum of the FIP provided to electricity producers in the carbon era to encourage the decarbonation of the electric mix, and the subsidy provided in both eras for the internalization of the learning by doing externality:

$$\sigma_{tot}(t) = \sigma(t) + \frac{\Sigma(t)}{\phi} = \underbrace{\zeta_1(t) - \widetilde{\tau}}_{\text{carbon pricing gap}} + \frac{1}{\phi}\underbrace{\left(-\frac{c_1'(Y(t))}{c_2}(\mu_d(t) - c_1(Y(t))\right)}_{\text{LbD externality subsidy}} \tag{34}$$

5.2 Calibration

We assume classically a CRRA utility function:

$$u(e) = \gamma\frac{e^{1-\frac{1}{\epsilon}}}{1-\frac{1}{\epsilon}}, \ \gamma > 0, \ \epsilon > 0, \ \epsilon \neq 1$$

and the following form for the marginal investment cost at the origin:

$$c_1(Y) = c_1 Y^{-\beta}$$

Let us define the learning rate ξ as the decrease of the cost $c_1(Y)$ taking place for every doubling of the green capacity: $-\xi = (c_1(2Y)/c_1(Y)) - 1$. With the above specification of the $c_1(Y)$ function we obtain $\beta = -\frac{\ln(1-\xi)}{\ln 2}$.

We do not pretend to calibrate our extremely stylized model to represent exhaustively and precisely the European energy transition. We apply our framework to the case of the European Union for illustrative purposes and to have a sense of the orders of magnitude.

Our simulations rely on two assumptions concerning the optimal energy transition policy. First, the long run total final energy consumption is equal to its initial level. Second, the initial optimal carbon value is 80 €/tCO_2. With these assumptions, the carbon budget that the European Union implicitly internalizes is 36.15% below its per capita share of the world carbon budget of 1,230 GTCO_2eq compatible with a 50% probability of limiting global warming to 2°C (Friedlingstein et al., 2022) and the optimal energy transition is achieved in 29 years.

The parameters chosen in the literature and the calibrated values are given in Table 1.

Parameters			
Discount rate	ρ	3%	literature
Green capital depreciation rate	δ	3%	literature
Price elasticity of electricity demand	ϵ	0.5	Labandeira et al. (2017)
Learning rate	ξ	20%	Rubin et al. (2015)
Load factor × number of hours per year	ϕ	0.34×8760 TWh/GW	data
Emission coefficient of fossils		0.604603 MtCO_2/MWh	data
Initial values			
Green capacity	Y_0	522.826 GW	data
Carbon intensive energy consumption	$x(0)$	9,500 TWh	data
Fossil unit production cost	c_x	60 €/MWh	data
Calibrated values			to fit
Utility scale parameter	γ	$1.467 \cdot 10^{10}$	$p_e(0) = 110$ €/MWh
Investment cost function parameter	c_1	58,089.8	initial average inv. cost = 8097 M€/
Capital adjustment cost parameter	c_2	22.444	$e^* = e(0)$ TWh
Carbon budget	$\overline{X}$	144,491 TWh	initial carbon value = 80 €/tCO_2

Table 1: Parameters, initial values and calibrated values

5.3 Results

Table 2 and Figure 2 illustrate the optimal trajectory, as well as the second best trajectories with two levels of the constant carbon tax, "large" and "small" with overshooting of its long run value by the green capital accumulated in the carbon era. Remember that "large" (resp. "small") refers to a carbon tax larger (resp. smaller) than the initial second best carbon value, which depends itself on the carbon tax.

Table 2 presents the main results. It displays in particular the *welfare cost of acceptability*, defined as the welfare loss at the second best compared to the optimum, measured by the equivalent constant additional electricity consumption that should be given to households to make them indifferent. With society's net surplus defined by:

$$W = \int_0^\infty e^{-\rho t} \left[u(e(t)) - c_x x(t) - C(I(t), Y(t))\right] dt$$

It is the welfare cost of acceptability reads:

$$w \equiv \left(1 + \frac{W^\circ - W^\sharp}{\int_0^\infty e^{-\rho t} u(e^\sharp(t))dt}\right)^{\frac{1}{1-\frac{1}{\epsilon}}} - 1 \tag{35}$$

Table 2 also display the cost of the second best policy in terms of public budget balance, defined as the present value of additional lump sum taxes or transfers that are levied on households in percentage of the present value of electricity consumption, over the carbon era:

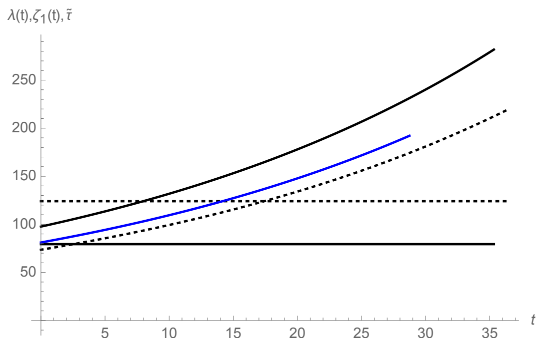
$$b \equiv \frac{\int_0^{\widetilde{T}^\sharp} e^{-\rho t}(\widetilde{\tau} x^\sharp(t) - \sigma(t) Y^\sharp(t))dt}{\int_0^{\widetilde{T}^\sharp} e^{-\rho t}(p_e^\sharp(t) e^\sharp(t))dt} = \frac{\int_0^{\widetilde{T}^\sharp} e^{-\rho t}\mathcal{T}(t)dt}{\int_0^{\widetilde{T}^\sharp} e^{-\rho t}(p_e^\sharp(t) e^\sharp(t))dt} \tag{36}$$

	$\lambda(0)$	$\widetilde{\tau}$	$\zeta_1(0)$	T_0	$\widetilde{T}$	$w(\%)$	$b(\%)$
First best	80				29		
Second best, "large" $\widetilde{\tau}$		125	73	17	37	2.1	2.2
Second best, "small" $\widetilde{\tau}$		80	100		35	3.0	-35.4

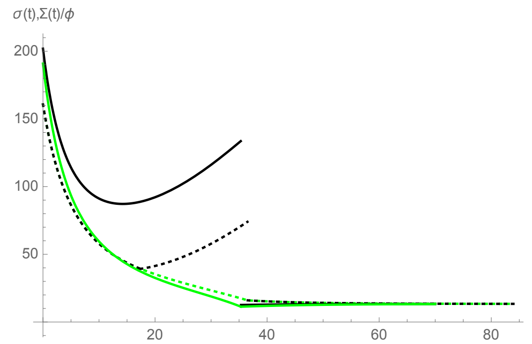
Table 2: First best and second best with constant carbon tax and FIP

With a "small" constant carbon tax, the FIP has to be very large to fill the carbon pricing gap. This policy fosters a very fast accumulation of green capital. Incidentally, electricity consumption is at each date higher than optimal, and electricity price lower. This result highlights the potential complementary role of consumption taxes. The policy based on very large subsidies to carbon-free energy sources is very costly in terms of welfare loss, and hugely costly for public finance. Fossil phase out is delayed. When it is achieved, in the clean era, electricity consumption that has been maintained at an artificially high value by the subsidies, decreases while its price increases.

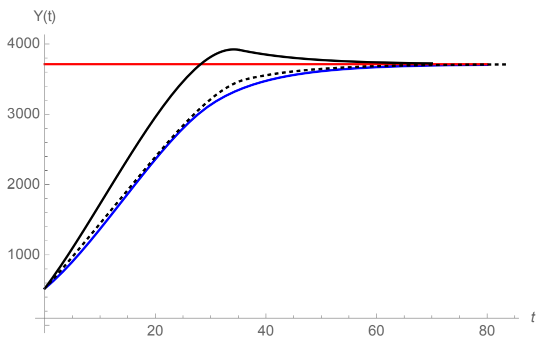
With a "large" constant carbon tax there is a first phase where the FIP is zero. Thus, in the beginning, the only subsidy is the one that internalizes the learning by doing externality. Then, only a small FIP is required to fill the carbon pricing gap. The resulting accumulation of green capital is slightly faster than what it is at the optimum. The climate policy generates a public



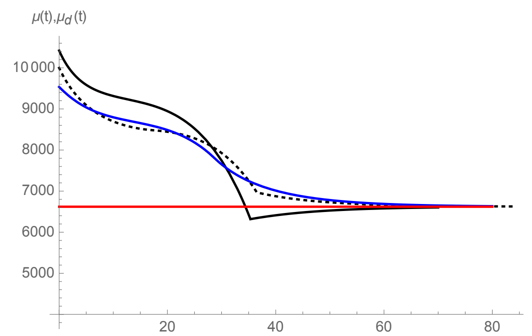
First best carbon value, effective carbon tax
and second best carbon value (€/tCO_2)



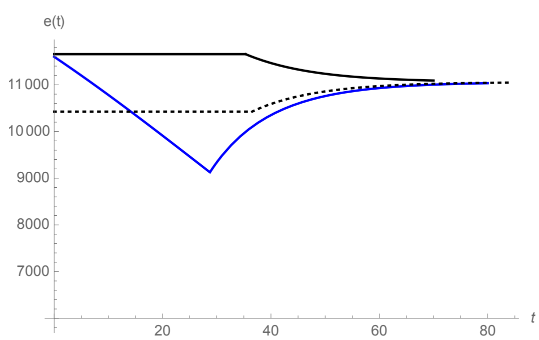
Total subsidies (black) and
learning by doing subsidies only (green)



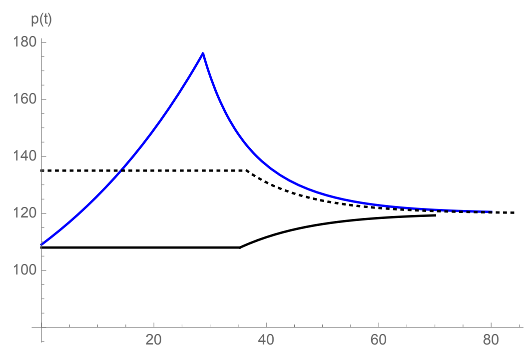
Green capital (GW)



First best social value of green capital and
second best private value of green capital



Electricity consumption (TWh)



Electricity price (€/MWh)

Figure 2: Optimal (blue) versus second best policy with constant carbon tax and FIP, for a large tax (black, dashed) and a small tax with overshooting (black, plain).

budget surplus, so that households receive lump sum transfers. However, the welfare loss is still large.

Clearly, the situation where the regulator puts in place a “large” constant carbon tax is better. But it is difficult to believe that if he cannot commit to the optimal carbon tax for political economy reasons, he would be able to put in place a higher carbon tax early on. Hence, the case closer to the public concern seem to be the one of a “small” carbon tax. As we have seen, it entails strong distortions in terms of investment, implying large inefficiencies and huge costs.

6 Conclusion

Confronted with political opposition to the implementation of an efficient direct carbon pricing, policy makers have been relying on a set of alternative policy interventions, including in particular subsidies to the production of carbon-free electricity (feed-in tariffs, feed-in premiums, renewable portfolio standards), subsidies to investment in specialized capital (tax credits, rebates on credit costs), as well as command and control regulation, or energy consumption taxes.

In this paper we explore the possibility of implementing a climate policy without relying on a carbon tax that increases at the optimal rate. We compare the performance in terms of welfare and related metrics the use of alternative patterns of policy tools. We do this in a dynamic macroeconomic model where the climate policy aims at using efficiently a given amount of cumulative carbon emissions.

We find that it could be possible to design two policy instruments, in such a way to replicate the same outcome as under optimal carbon pricing. This is the case when a tax is levied on electricity consumption, while a feed-in premium is paid to carbon-free electricity, or investment in green production and storage capacity is directly subsidized. However, such a tax on electricity consumption would be increasing as well, making it hardly politically more feasible than an efficient carbon tax.

In the case that climate policy should rely on subsidizing carbon-free sources to complement a small constant carbon tax, we find that meeting the climate target is feasible, but at a huge cost. The subsidies have to be very large, in order to foster green investment at a fast pace, resulting in important investment costs, weighing on households’ budgets. Overall, larger investment costs and deformed consumption paths together reduce welfare. The key problem is that subsidies to carbon-free technologies do not tackle directly the issue of limiting fossil resource use early on. They can crowd out fossils from the market, but the second best policy implies some strong and costly distortions such as boosting energy consumption, during the early phase when it should be falling.

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Appendices

A Decentralized equilibrium

The program of the representative household is $\forall t \geq 0$:

$$\begin{aligned} \max_{e(t),z(t)} \quad & \int_0^\infty e^{-\rho t} \left[z(t) + u\left(e(t)\right)\right] dt \\ \text{s.t.} \quad & \dot{a}(t) = r(t)a(t) - z(t) - \left(p_e(t) + \theta(t)\right) e(t) + \Pi_x(t) + \Pi_e(t) + \mathcal{T}(t) \\ & a(0) = a_0 \text{ given} \end{aligned}$$

where a is the household's financial wealth and a_0 its endowment, r is the real rate of return on financial wealth as well as the interest rate at which the household can borrow, while Π_x and Π_e are the profits of the fossil resource producers and of the electricity producers respectively. The households' optimal saving behavior ensures that $\forall t \geq 0$ $r(t) = \rho$, while his behavior on the electricity market is characterized by the demand function $e(p_e(t) + \theta(t))$ (equation 6).

Defining $R(t) \equiv \int_0^t r(u)du$, the program of the power producer is:

$$\begin{aligned} \max_{x(t),I(t)} \quad & \int_0^\infty e^{-R(t)} \left[p_e(t)x(t) + \left(p_e(t) + \sigma(t)\right) \phi Y(t) - \left(p_x(t) + \tau(t)\right) x(t) - C(I(t)) + s(t)I(t)\right] dt \\ \text{s.t.} \quad & (3),\ x(t) \geq 0,\ I(t) \geq 0 \text{ and } Y(0) = Y_0 \text{ given} \end{aligned}$$

Let us write the Hamiltonian and the Lagrangian of this program, denoting μ_d the shadow price of green capacity and ω_I the Lagrange multiplier associated to the investment positivity constraint:

$$\mathcal{H}^u = p_e x + (p_e + \sigma)\phi Y - (p_x + \tau)x - C(I) + sI + \mu_d(I - \delta Y)$$

$$\mathcal{L}^u = \mathcal{H}^u + \omega_I I$$

The Hamiltonian is linear in x. Equations (7) and (8) follow.

The first order condition characterizing the investment decision is:

$$C'(I) = \mu_d + s + \omega_I$$

When $I > 0$, $\omega_I = 0$ and the condition reduces to (9). The investment positivity constraint is binding when $C'(0) > \mu_d$. The evolution of μ_d over time is given by equation (10).

In equilibrium, when the fossil resource is used to produce power, (10) can be written using the equality of the interest and discount rates and equations (7) and (11), to get equation (12). The equilibrium on the electricity market is obtained by combining (6), (7) and (11), to get equation (13).

The condition of existence of the phase when the fossil resource is used is $x = e(\tau + \theta) - \phi Y > 0$: residual demand has to be positive. To put it differently, the marginal utility of electricity consumption from renewable sources $u'(\phi Y)$ has to be higher than the price of electricity $\tau + \theta$. If the path of green capital Y is weakly increasing over time, $u'(\phi Y)$ is weakly decreasing. Then, if $\tau + \theta$ is weakly increasing, there exists a unique date T at which $u'(\phi Y(T)) = \tau(T) + \theta(T)$. The fossil resource is used before T, and is not used after T any more. Notice that μ_d, the costate of green capacity, is necessarily continuous at T.

Let us finally derive the regulator's objective function. Using the equation of evolution of household's wealth, the profits and the public budget constraint, and the transversality condition, we obtain:

$$a_0 = \int_0^\infty e^{-\rho t} \left[z(t) + (p_e(t) + \theta(t))e(t) - \Pi_e(t) - \mathcal{T}(t) \right] dt = \int_0^\infty e^{-\rho t} \left[z(t) + C(I(t)) \right] dt$$

Therefore

$$\int_0^\infty e^{-\rho t} z(t) dt = a_0 - \int_0^\infty e^{-\rho t} C(I(t)) dt$$

and, since a_0 is given, the regulator's objective function reads:

$$W = \int_0^\infty e^{-\rho t} \left[u(x(t) + \phi Y(t)) - C(I(t)) \right] dt$$

B Optimal path

In order to simplify notation, we do not write the dependency of variables on time.

The planner maximizes society's net surplus W defined in equation (17), subject to the technology and resource constraints (1)-(3), as well as the non-negativity and carbon budget constraints, for given initial conditions (4). The Hamiltonian and the Lagrangian of the problem are:

$$\mathcal{H} = u(x + \phi Y) - C(I) - \lambda x + \mu(I - \delta Y)$$

$$\mathcal{L} = \mathcal{H} + \omega_x x + \omega_I I + \omega_X(\overline{X} - X)$$

The first order conditions read:

$$u'(x + \phi Y) = \lambda - \omega_x \tag{B.1}$$

$$C'(I) = \mu + \omega_I \tag{B.2}$$

$$\dot{\lambda} = \rho\lambda - \omega_X \tag{B.3}$$

$$\dot{\mu} = (\rho + \delta)\mu - \phi u'(x + \phi Y) \tag{B.4}$$

According to equation (B.2), investment is nil if and only if $C'(0) > \mu$. To avoid unnecessary complexities, we restrict ourselves to the case where Y_0 is sufficiently small for investment in green capital to take place at the beginning of the planning horizon ($C'(0) < \mu(0)$). It does not preclude the fact that investment may become nil later on.

In the carbon era, $x > 0, \omega_x = 0, \omega_X = 0$. Then (B.3) is (19), and (B.1) implies (20) for $t < T$, and also the path of fossil resource use :

$$x^\circ(t) = e\left(\lambda^\circ(0)e^{\rho t}\right) - \phi Y^\circ(t) \tag{B.5}$$

The carbon era comes to an end at T such that $x^\circ(T) = 0$, $Y^\circ(T) = \frac{1}{\phi}e(\lambda^\circ(0)e^{\rho T})$ and the carbon budget is exhausted: $\int_0^T x^\circ(t)dt = \overline{X}$ with (B.5).

In the clean era, $x = 0$, $\omega_x > 0$, $\omega_X > 0$. Substituting for $x = 0$ into (B.4) gives (20) for $t \geq T$. The differential system (18)–(20) defines the steady state (23), which is saddle path stable.

C Second best, FIP

C.1 Clean era

The proof of Lemma 1 is trivial. At the first best, the clean era sharpts at date T° with a green capital stock $Y^\circ(T^\circ) = e(\lambda^\circ(T^\circ))/\phi$ (equation (22)), whereas at the second best it sharpts at date $\widetilde{T}$ with a green capital stock $Y(\widetilde{T}) = e(\widetilde{\tau})/\phi$ (equation (14) with $\tau(t) = \widetilde{\tau}$ and $\theta(t) = 0$). This proves (ii). (iii) is directly derived from the definition of $Y(\widetilde{T})$: $Y(\widetilde{T}) > Y^* \Leftrightarrow e(\widetilde{\tau})/\phi > Y^* \Leftrightarrow \widetilde{\tau} < u'(\phi Y^*)$.

C.2 Carbon era

(i) derives directly from equation (6); since the consumer price of electricity is $\widetilde{\tau}$, demand for electricity is at each date in the carbon era is equal to $e(\widetilde{\tau})$.

To prove (ii) we derive the optimality condition, related to the choice of the optimal date of the switch to the clean era $\widetilde{T}$.

The regulator's welfare function is:

$$W \equiv \int_0^{\widetilde{T}} e^{-\rho t}\left[u(e(\widetilde{\tau})) - C(I(\mu_d(t)))\right]dt + \int_{\widetilde{T}}^{\infty} e^{-\rho t}\left[u(\phi Y(t)) - C(I(\mu_d(t)))\right]dt$$

The regulator chooses $\widetilde{T}$ optimally. Therefore

$$\frac{\partial W}{\partial \widetilde{T}} = e^{-\rho \widetilde{T}}\left[u(e(\widetilde{\tau})) - C(I(\mu_d(\widetilde{T}^-)))\right] - e^{-\rho \widetilde{T}}\left[u(\phi Y(\widetilde{T})) - C(I(\mu_d(\widetilde{T}^+)))\right] = 0$$

As $e(\widetilde{\tau}) = \phi Y(\widetilde{T})$, it proves that $\mu_d(\widetilde{T}^-) = \mu_d(\widetilde{T}^+)$, that is the continuity of the private value of green capital at the date of the switch to the clean era.

To prove the other parts of the proposition we must solve the regulator's program that reads, given the reaction functions of the agents:

$$
\begin{aligned}
&\max_{\sigma(.)} \int_0^{\widetilde{T}} e^{-\rho t} \left[u(e(\widetilde{\tau})) - C(I(\mu_d(t)))\right] dt \\
&\qquad \dot{X}(t) = e(\widetilde{\tau}) - \phi Y(t) \\
&\qquad \dot{Y}(t) = I(\mu_d(t)) - \delta Y(t) \\
&\qquad \dot{\mu}_d(t) = (\rho + \delta)\mu_d(t) - \phi(\widetilde{\tau} + \sigma(t)) \\
&\qquad \sigma(t) \geq 0,\ X(t) \leq \overline{X} \\
&\qquad X(0) = 0\ ,\ Y(0) = Y_0\ ,\ Y(\widetilde{T}) = \frac{1}{\phi} e(\widetilde{\tau}),\ \mu_d(\widetilde{T}^+) = \mu_d(\widetilde{T}^-),\ \mu_d(0)\ \text{ free}
\end{aligned}
$$

The associated Hamiltonian is:

$$
\mathcal{H} = u(e(\widetilde{\tau})) - C(I(\mu_d)) - \zeta_1(e(\widetilde{\tau}) - \phi Y) + \zeta_2(I(\mu_d) - \delta Y) + \zeta_3\left[(\rho + \delta)\mu_d - \phi(\widetilde{\tau} + \sigma)\right]
$$

where ζ_1 is the second best carbon value, ζ_2 the second best value of green capital and ζ_3 the costate of μ_d.

Concerning the first order condition on the FIP, notice that the Hamiltonian is linear in σ, and that $\partial\mathcal{H}/\partial\sigma = -\zeta_3\phi$. Therefore, the condition yields:

$$
\sigma(t) \begin{cases} > 0 & \text{if} \quad \zeta_3 = 0 \\ = 0 & \text{if} \quad \zeta_3 > 0 \end{cases} \tag{C.1}
$$

The other first order conditions are:

$$
\dot{\zeta}_1 = \rho\zeta_1 \tag{C.2}
$$
$$
\dot{\zeta}_2 = (\rho + \delta)\zeta_2 - \zeta_1\phi \tag{C.3}
$$
$$
\dot{\zeta}_3 = -\delta\zeta_3 + I'(\mu_d)(\mu_d - \zeta_2) \tag{C.4}
$$

Equation (C.2) shows that ζ_1 obeys the Hotelling rule. This proves (iii).

The proof of (iv) a. is trivial.

We now prove (iv) b.–f.

Suppose that we are in the interior case, $\sigma > 0$, on a given, non-degenerate interval of time. Then $\zeta_3 = 0$ all along this interval (see conditions (C.1)), and equation (C.4) yields $\mu_d = \zeta_2$, meaning that thanks to the FIP, the private and social shadow values of green capital are the same. Then equations (12) and (C.3) imply (26).

The lower bound of the second best carbon value is $\zeta_1(0)$. When $\widetilde{\tau} < \widehat{\tau} = \zeta_1(0)|_{\widehat{\tau}}$, the FIP is positive all along the carbon area. When $\widetilde{\tau} > \widehat{\tau}$, there is at the beginning of the horizon a period when the FIP should be negative, which is unfeasible. The phase where $\sigma(t) > 0$ is necessarily preceded by a phase where, because the carbon tax is higher than the initial second best carbon value, $\zeta_3(t) > 0$ and $\sigma(t) = 0$. The length of this phase is increasing in $\widetilde{\tau}$. At the limit, this phase lasts until fossil phase out, which occurs for $\widetilde{\tau} = \overline{\tau} = \zeta_1(0)e^{\rho\widetilde{T}}|_{\overline{\tau}}$. This proves (iv) b. and c.

We have seen in Lemma 1 that the value of the carbon tax may entail optimal overshooting of the green capital, compared to its long term value ($\widetilde{\tau} < \tau^{ov}$). The limit case of this situation is when this overshooting is so large that disinvesting in the clean era would be optimal, which is not possible according to our assumptions. The limit is provided by the equality of the private shadow value of green capital for electricity producers and the marginal investment cost for $I = 0$ at the beginning at the clean era: $\mu_d(\widetilde{T}^+) = C'(0)$. This limit translates into an upper bound for $Y(\widetilde{T})$, which in turn translates into a lower bound $\underline{\tau}^{ov}$ for the carbon tax, and an upper bound $\sigma_{max} = \sigma(\widetilde{T})|_{\widetilde{\tau}=\underline{\tau}^{ov}}$. This proves (iv) d–f.

Finally, (v) is true because μ_d is non-controllable, in the sense of Xie (1997) and Dockner et al. (2000), chapter 5. In the case where the FIP is always positive, the non-controllability od μ_d stems from the fact that its costate ζ_3 is identically 0 on the interval $[0, \widetilde{T}]$. Indeed, when $\zeta_3(0) = 0$ is a necessary condition for optimality and $\zeta_3(t) \neq 0$ for $t > 0$, the regulator re-computing its optimal policy at a later date $t' > 0$, would want to set $\zeta_3(t') = 0$, hence the time-inconsistency of his policy. This cannot happen here. In the cases where the FIP is nil at the beginning of the horizon, $\zeta_3(0) \neq 0$, and therefore μ_d is still non-controllable.

We provide in what follows a thorough analysis of the dynamic systems characterizing the carbon era in the case of quadratic adjustment costs

$$C(I) = c_1 I + \frac{c_2}{2} I, \quad c_1 > 0,\ c_2 > 0$$

as well as the proofs concerning the sign of ζ_3.

C.2.1 "Small" carbon tax, $\widetilde{\tau} \in [\underline{\tau}, \widehat{\tau}]$

In this case, $\widetilde{\tau} < \zeta_1(0)$, $\zeta_3(t) = 0$ and $\sigma(t) \in [0, \sigma_{max}]$ $\forall t \in [0, \widetilde{T}]$.

The dynamic system to be solved before $\widetilde{T}$ is:

$$\begin{aligned}
\dot{Y} &= \frac{1}{c_2}(\mu_d - c_1) - \delta Y \\
\dot{\mu}_d &= (\rho + \delta)\mu_d - \phi(\widetilde{\tau} + \sigma) \\
\sigma &= \zeta_1 - \widetilde{\tau} \\
\dot{\zeta}_2 &= (\rho + \delta)\zeta_2 - \phi\zeta_1 \\
\dot{\zeta}_1 &= \rho\zeta_1
\end{aligned}$$

It reduces to a differential system of two equations and two unknowns, Y and μ_d:

$$\dot{Y} = \frac{1}{c_2}(\mu_d - c_1) - \delta Y \tag{C.5}$$

$$\dot{\mu}_d = (\rho + \delta)\mu_d - \phi\zeta_1(0)e^{\rho t} \tag{C.6}$$

This system yields the following second order linear differential equation in Y:

$$\ddot{Y} - \rho\dot{Y} - \delta(\rho + \delta)Y = (\rho + \delta)\frac{c_1}{c_2} - \frac{\phi}{c_2}\zeta_1(0)e^{\rho t}$$

The solution of this equation is the sum of the solution of the homogeneous equation and of a particular solution of the non-homogeneous equation. The homogeneous equation has two real roots of opposite sign, $\rho + \delta$ and $-\delta$. A particular solution of the non-homogeneous equation is found by the method of undetermined coefficients. We guess that the solution has the form $\alpha + \beta e^{\rho t}$. Then

$$-\delta(\rho + \delta)(\alpha + \beta e^{\rho t}) = (\rho + \delta)\frac{c_1}{c_2} - \frac{\phi}{c_2}\zeta_1(0)e^{\rho t}$$

which yields:

$$\alpha = -\frac{c_1}{c_2\delta} \quad \text{and} \quad \beta = \frac{\phi}{c_2\delta(\rho + \delta)}\zeta_1(0)$$

The solution is therefore

$$Y(t) = A_1 e^{(\rho+\delta)t} + A_2 e^{-\delta t} + \frac{\phi}{c_2\delta(\rho + \delta)}\zeta_1(0)e^{\rho t} - \frac{c_1}{c_2\delta} \tag{C.7}$$

where A_1 and A_2 are constants of integration.

Using (C.7) we can recover $\mu_d(t)$:

$$\mu_d(t) = c_2(\dot{Y}(t) + \delta Y(t)) + c_1 = A_1 c_2(\rho + 2\delta)e^{(\rho+\delta)t} + \frac{\phi}{\delta}\zeta_1(0)e^{\rho t} \tag{C.8}$$

The four unknowns are A_1, A_2, $\zeta_1(0)$ and $\widetilde{T}$. The four conditions allowing to compute them are the three boundary conditions (initial and terminal conditions on Y plus the continuity of μ_d at $\widetilde{T}$)

and the carbon budget exhaustion equation:

$$\begin{aligned} &Y(0) = Y_0 \\ &Y(\widetilde{T}) = \frac{e(\widetilde{\tau})}{\phi} \\ &\mu_d(\widetilde{T}^-) = \mu_d(\widetilde{T}^+) \\ &\overline{X} = e(\widetilde{\tau})\widetilde{T} - \phi \int_0^{\widetilde{T}} Y(t)dt \end{aligned} \tag{C.9}$$

$\mu_d(\widetilde{T}^+)$, the private value of green capital corresponding to an initial condition $Y(\widetilde{T})$ on the saddle path of the clean era, depends on the steady state values Y^* and μ^*, on $Y(\widetilde{T})$ and on the parameters.

C.2.2 "Large" carbon tax, $\widetilde{\tau} \in [\widehat{\tau}, \overline{\tau}]$

In this case, $\widetilde{\tau} > \zeta_1(0)$, $\zeta_3(t) > 0$ and $\sigma(t) = 0$ $\forall t \in [0, T_0]$, $\zeta_3(t) = 0$ and $\sigma(t) > 0$ $\forall t \in (T_0, \widetilde{T}]$.

The dynamic system to be solved for $t \in [0, T_0]$ is:

$$\dot{Y} = \frac{1}{c_2}(\mu_d - c_1) - \delta Y \tag{C.5}$$

$$\dot{\mu}_d = (\rho + \delta)\mu_d - \phi\widetilde{\tau} \tag{C.10}$$

It yields the following second order linear differential equation in Y:

$$\ddot{Y} - \rho\dot{Y} - \delta(\rho + \delta)Y = (\rho + \delta)\frac{c_1}{c_2} - \frac{\phi}{c_2}\widetilde{\tau}$$

which can be solved with the same method as in the previous case, to obtain:

$$Y(t) = B_1 e^{(\rho+\delta)t} + B_2 e^{-\delta t} + \frac{\phi}{c_2\delta(\rho + \delta)}\widetilde{\tau} - \frac{c_1}{c_2\delta} \tag{C.11}$$

where B_1 and B_2 are constants of integration.

Using (C.11) we can recover $\mu_d(t)$:

$$\mu_d(t) = B_1 c_2(\rho + 2\delta)e^{(\rho+\delta)t} + \frac{\phi}{\rho + \delta}\widetilde{\tau} \tag{C.12}$$

The solution of the dynamic system for $t \in (T_0, \widetilde{T}]$ is described by the same set of equations as in the case of a "small" carbon tax (equations (C.7) and (C.8)).

The seven unknowns are B_1, B_2, A_1, A_2, $\zeta_1(0)$, T_0 and $\widetilde{T}$. The seven conditions allowing to compute them are the four conditions (C.9), to which must be added the three following conditions of continuity at T_0:

$$\begin{aligned}\zeta_1(0)e^{\rho T_0} &= \widetilde{\tau} \\ Y(T_0^-) &= Y(T_0^+) \\ \mu_d(T_0^-) &= \mu_d(T_0^+)\end{aligned} \tag{A.9}$$

Using $\zeta_3(T_0) = 0$, equation (C.4) in the text integrates into:

$$\zeta_3(t) = \frac{1}{c_2}\int_t^{T_0} e^{\delta(s-t)}(\mu_d(s) - \zeta_2(s))ds$$

Before T_0 the social value of green capital ζ_2 is not equal to its private value μ_d, as it is the case after T_0. It is lower, which we are going to prove below. For them to be equal it would have been necessary to tax green capital instead of subsidizing it, in order to slow down its accumulation.

Equation (C.3) in the text integrates into:

$$\zeta_2(t) = \left(\zeta_2(0) - \frac{\phi}{\delta}\zeta_1(0)\right)e^{(\rho+\delta)t} + \frac{\phi}{\delta}\zeta_1(0)e^{\rho t}$$

Equation (C.12) may be written as:

$$\mu_d(t) = \left(\mu_d(0) - \frac{\phi}{\rho+\delta}\widetilde{\tau}\right)e^{(\rho+\delta)t} + \frac{\phi}{\rho+\delta}\widetilde{\tau}$$

We have $\mu_d(T_0) = \zeta_2(T_0)$ and $\zeta_1(0)e^{\rho T_0} = \widetilde{\tau}$. Therefore, at T_0,

$$\left(\zeta_2(0) - \frac{\phi}{\delta}\widetilde{\tau}e^{-\rho T_0}\right)e^{(\rho+\delta)T_0} + \frac{\phi}{\delta}\widetilde{\tau} = \left(\mu_d(0) - \frac{\phi}{\rho+\delta}\widetilde{\tau}\right)e^{(\rho+\delta)T_0} + \frac{\phi}{\rho+\delta}\widetilde{\tau}$$

i.e.

$$\mu_d(0) - \zeta_2(0) = \left(1 + \frac{\rho}{\delta}e^{-(\rho+\delta)T_0} - \left(1+\frac{\rho}{\delta}\right)e^{-\rho T_0}\right)\frac{\phi}{\rho+\delta}\widetilde{\tau}$$

Let $f(t) = 1 + \frac{\rho}{\delta}e^{-(\rho+\delta)t}$ and $g(t) = \left(1+\frac{\rho}{\delta}\right)e^{-\rho t}$. f and g are positive functions from $[0,+\infty)$ to $(0, 1+\frac{\rho}{\delta}]$, with $f(0) = g(0) = 1 + \frac{\rho}{\delta}$ and $\lim_{t\to\infty} f(t) = \lim_{t\to\infty} g(t) = 0$. Besides, $f'(t) = -\frac{\rho(\rho+\delta)}{\delta}e^{-(\rho+\delta)t}$ and $g'(t) = -\frac{\rho(\rho+\delta)}{\delta}e^{-\rho t} = f'(t)e^{\delta t}$. As f' and g' are negative, it implies that $g'(t) < f'(t)\ \forall t > 0$, meaning that the g function decreases faster than the f function, i.e. $g(t) < f(t)\ \forall t > 0$. We can then conclude that $\mu_d(0) - \zeta_2(0) > 0$.

We must finally prove that $\mu_d(t) - \zeta_2(t) > 0\ \forall t \in [0, T_0]$.

We have:

$$(\mu_d(t) - \zeta_2(t))\,e^{-(\rho+\delta)t} = \mu_d(0) - \zeta_2(0) - \frac{\phi}{\rho+\delta}\widetilde{\tau} + \frac{\phi}{\delta}\zeta_1(0) + \frac{\phi}{\rho+\delta}\widetilde{\tau}e^{-(\rho+\delta)t} - \frac{\phi}{\delta}\zeta_1(0)e^{-\delta t}$$

Replacing $\mu_d(0) - \zeta_2(0)$ by its expression obtained above and $\widetilde{\tau}$ by $\zeta_1(0)e^{\rho T_0}$ we obtain:

$$\begin{aligned}
\frac{e^{-(\rho+\delta)t}}{\phi\zeta_1(0)}\left(\mu_d(t) - \zeta_2(t)\right) &= \frac{1}{\delta(\rho+\delta)}\left(\delta + \rho e^{-(\rho+\delta)T_0} - (\rho+\delta)e^{-\rho T_0}\right)e^{\rho T_0} \\
&\quad - \frac{1}{\rho+\delta}e^{\rho T_0} + \frac{1}{\delta} + \frac{1}{\rho+\delta}e^{\rho T_0}e^{-(\rho+\delta)t} - \frac{1}{\delta}e^{-\delta t} \\
&= \frac{1}{\delta(\rho+\delta)}\left(\delta e^{\rho T_0} + \rho e^{-\delta T_0} - (\rho+\delta) - \delta e^{\rho T_0} + (\rho+\delta) + \delta e^{\rho T_0}e^{-(\rho+\delta)t} - (\rho+\delta)e^{-\delta t}\right) \\
&= \frac{1}{\delta(\rho+\delta)}\left(\rho(e^{-\delta T_0} - e^{-\delta t}) + \delta(e^{\rho(T_0-t)} - 1)e^{-\delta t}\right) \\
&= \frac{e^{-\delta t}}{\delta(\rho+\delta)}\left(\delta(e^{\rho(T_0-t)} - 1) - \rho(1 - e^{-\delta(T_0-t)}\right)
\end{aligned}$$

Therefore

$$\mu_d(t) - \zeta_2(t) = \frac{\phi\zeta_1(0)e^{\rho t}}{\delta(\rho+\delta)}\left(\delta(e^{\rho(T_0-t)} - 1) - \rho(1 - e^{-\delta(T_0-t)}\right) \;\; \forall t \in [0, T_0]$$

Let $F(t) = \delta(e^{\rho(T_0-t)} - 1) - \rho(1 - e^{-\delta(T_0-t)})$. We have $F(0) = \delta(e^{\rho T_0} - 1) - \rho(1 - e^{-\delta T_0}) > 0$ since $\mu_d(0) - \zeta_2(0) > 0$, $F(T_0) = 0$, and $F'(t) = -\delta\rho(e^{\rho(T_0-t)} - e^{-\delta(T_0-t)}) < 0$. F is monotonously decreasing from a positive value to 0. Therefore $F(t) > 0 \; \forall t \in [0, T_0]$. We can then conclude that $\mu_d(t) - \zeta_2(t) > 0$.

C.2.3 'Binding investment irreversibility constraint, $\widetilde{\tau} < \underline{\tau}^{ov}$

In this case, $\widetilde{\tau} < \zeta_1(0)$, $\zeta_3(t) = 0$ and $\sigma(t) \in]0, \sigma_{max}[\; \forall t \in [0, T_{max}[$ and $\sigma(t) = \sigma_{max} \; \forall t \in [T_{max}, \widetilde{T}]$.

The dynamic system to be solved before T_{max} is (C.5)–(C.6). After T_{max} it is (C.5)–(C.10), where $\widetilde{\tau}$ is replaced by $\widetilde{\tau}+\sigma_{max}$ in the second equation. Boundary conditions are the four conditions (C.9), to which must be added the three following conditions of continuity at T_{max}:

$$\begin{aligned}
\zeta_1(0)e^{\rho T_{max}} &= \widetilde{\tau} + \sigma_{max} \\
Y(T_{max}^-) &= Y(T_{max}^+) \\
\mu_d(T_{max}^-) &= \mu_d(T_{max}^+)
\end{aligned} \tag{A.10}$$

In order to compute the value of $\underline{\tau}^{ov}$ we linearize the dynamic system characterizing the evolution of the economy in the clean phase around the steady state. It yields:

$$\mu_d(t) = \mu^* + c_2(\omega+\delta)(Y(\widetilde{T}) - Y^*)e^{\omega(t-\widetilde{T})}$$

with

$$\omega = \frac{1}{2}\left[\rho - \sqrt{(\rho+2\delta)^2 + \frac{4\gamma}{c_2 Y^{*2}}}\right] < 0$$

and

$$Y^* = \frac{\mu^* - c_1}{\delta c_2}$$

Then:

$$\begin{aligned}\mu_d(\widetilde{T}^+) - c_1 &= \mu^* - c_1 + c_2(\omega+\delta)(Y(\widetilde{T}) - Y^*)\\ &= c_2(\omega+\delta)Y(\widetilde{T}) - c_2\omega Y^*\end{aligned}$$

and

$$\mu_d(\widetilde{T}^+) \geq c_1 \Leftrightarrow (\omega+\delta)Y(\widetilde{T}) \geq \omega Y^* \Leftrightarrow (\omega+\delta)\frac{u'^{-1}(\widetilde{\tau})}{\phi} \geq \omega Y^*$$

We have to determine the sign of $\omega + \delta$.

$$\begin{aligned}\omega = \frac{1}{2}\left[\rho - \sqrt{(\rho+2\delta)^2 + \frac{4\gamma}{c_2 Y^{*2}}}\right] \Rightarrow \rho - 2\omega = \sqrt{(\rho+2\delta)^2 + \frac{4\gamma}{c_2 Y^{*2}}}\\ \Rightarrow (\rho - 2\omega)^2 = (\rho+2\delta)^2 + \frac{4\gamma}{c_2 Y^{*2}} \Rightarrow -4(\rho+\delta-\omega)(\omega+\delta) = \frac{4\gamma}{c_2 Y^{*2}}\end{aligned}$$

which shows that $\omega + \delta < 0$.

We have:

$$\mu_d(\widetilde{T}^+) \geq c_1 \Leftrightarrow u'^{-1}(\widetilde{\tau}) \leq \frac{\omega}{\omega+\delta}\phi Y^* \Leftrightarrow \widetilde{\tau} \geq u'\left(\frac{\omega}{\omega+\delta}\phi Y^*\right) = \underline{\tau}^{ov}$$

As $\omega < 0$, $\frac{\omega}{\omega+\delta} > 1$, implying that $u'\left(\frac{\omega}{\omega+\delta}\phi Y^*\right) < u'\left(\phi Y^*\right)$. Therefore $\underline{\tau}^{ov} < \tau^{ov}$.

Finally, $\sigma_{max} = \sigma(\widetilde{T}^-)|_{\underline{\tau}^{ov}}$.

D Balanced budget

We show in this appendix that when a balanced budget is imposed, to each level of the constant carbon tax corresponds one and only one carbon budget, meaning that when the regulator wants to respect a given carbon budget he does not have any choice regarding the level of the carbon tax.

The carbon era is characterized by:

$$\begin{cases}\dot{Y}(t) &= I(\mu_d(t)) - \delta Y(t)\\ \dot{\mu}_d(t) &= (\rho+\delta)\mu_d(t) - \frac{\widetilde{\tau}e(\widetilde{\tau})}{Y(t)}\end{cases}$$

It is easy to show that the steady state equilibrium of this system is a saddle point, and to compute the saddle branch, parametrized by $\widetilde{\tau}$. Then, sharpting from an initial green capacity Y_0 of shadow value $\mu_d(0)$ on the stable branch, green capital increases, its shadow value decreases, until date $\widetilde{T}$ when $Y(\widetilde{T}) = \frac{1}{\phi}e(\widetilde{\tau})$. Having computed the green capital path, we deduce fossil use in the carbon era, $x(t) = e(\widetilde{\tau}) - \phi Y(t)$, parametrized by $\widetilde{\tau}$ as well. Then, for each level of the carbon budget, it is possible to compute the carbon tax ensuring that cumulative emissions are equal to this carbon budget. It is given by

$$\int_0^{\widetilde{T}} x(t)dt = e(\widetilde{\tau})\widetilde{T} - \phi \int_0^{\widetilde{T}} Y(t)dt = \overline{X}$$

E Comparing subsidies and FIP

The regulator's welfare function is:

$$W \equiv \int_0^{\widetilde{T}} e^{-\rho t} \left[u(e(\widetilde{\tau})) - C(I(\mu_d(t) + s(t)))\right] dt + \int_{\widetilde{T}}^{\infty} e^{-\rho t} \left[u(\phi Y(t)) - C(I(\mu_d(t)))\right] dt$$

The regulator chooses $\widetilde{T}$ optimally. Therefore

$$\frac{\partial W}{\partial \widetilde{T}} = e^{-\rho \widetilde{T}} \left[u(e(\widetilde{\tau})) - C(I(\mu_d(\widetilde{T}^-) + s(\widetilde{T}^-))\right] - e^{-\rho \widetilde{T}} \left[u(\phi Y(\widetilde{T})) - C(I(\mu_d(\widetilde{T}^+)))\right] = 0$$

As $e(\widetilde{\tau}) = \phi Y(\widetilde{T})$, the condition reduces to $\mu_d(\widetilde{T}^-) + s(\widetilde{T}^-) = \mu_d(\widetilde{T}^+)$. The private value of green capital being continuous at $\widetilde{T}$, it proves that $s(\widetilde{T}^-) = 0$: contrary to the FIP, that jumps at $\widetilde{T}$ from a positive value to zero, the subsidy to green investment is continuous at $\widetilde{T}$.

For a given $\widetilde{\tau}$, the regulator's program in the carbon era reads:

$$\begin{aligned}
&\max_{s(.)} \int_0^{\widetilde{T}} e^{-\rho t} \left[u(e(\widetilde{\tau})) - C(I(\mu_d(t) + s(t)))\right] dt \\
&\dot{X}(t) = e(\widetilde{\tau}) - \phi Y(t) \\
&\dot{Y}(t) = I(\mu_d(t) + s(t)) - \delta Y(t) \\
&\dot{\mu}_d(t) = (\rho + \delta)\mu_d(t) - \phi\widetilde{\tau} \\
&X(t) \le \overline{X},\ s(t) \ge 0 \\
&X(0) = 0\ ,\ Y(0) = Y_0\ ,\ Y(\widetilde{T}) = \frac{1}{\phi}e(\widetilde{\tau}),\ \mu_d(\widetilde{T}^+) = \mu_d(\widetilde{T}^-),\ \mu_d(0)\ \text{ free}
\end{aligned}$$

The Hamiltonian of the program is:

$$\mathcal{H} = u(e(\widetilde{\tau})) - C(I(\mu_d + s)) - \zeta_1(e(\widetilde{\tau}) - \phi Y) + \zeta_2(I(\mu_d + s) - \delta Y) + \zeta_3\left[(\rho + \delta)\mu_d - \phi\widetilde{\tau}\right]$$

and the Lagrangian is:

$$\mathcal{L} = \mathcal{H} + \omega_s s$$

The first order conditions are:

$$(\zeta_2 - C'(I))I'(\mu_d + s) = \omega_s \tag{E.1}$$

$$\dot{\zeta}_1 = \rho\zeta_1 \tag{E.2}$$

$$\dot{\zeta}_2 = (\rho + \delta)\zeta_2 - \phi\zeta_1 \tag{E.3}$$

$$\dot{\zeta}_3 = -\delta\zeta_3 \tag{E.4}$$

As $C'(I) = \mu_d + s$, condition (E.1) shows that the subsidy during the carbon era is, when it is positive, equal to the difference between the private and social shadow values of renewable capacity (equation (29)). Besides, differentiating (E.1) with respect to time and using conditions (E.3) and (12) with $\tau(t) = \widetilde{\tau}$ and $\sigma(t) = 0$, yields:

$$\dot{s} = \dot{\zeta}_2 - \dot{\mu}_d = (\rho + \delta)s - \phi(\zeta_1 - \widetilde{\tau})$$

which can be integrated forward into (30). This proves (i).

As in the case of a FIP, ζ_3 is non-controllable, which ensures the time consistency of the regulator's policy. Indeed, thanks to equation (E.4), if $\zeta_3(0) = 0$ then $\zeta_3(t)$ is identically 0 all along the carbon era.

A straightforward comparison of the dynamic systems characterizing the evolution of the economy in the FIP case and in the subsidy to investment case shows that the two instruments are strictly equivalent. Both systems coalesce into a dynamic system in Y and ζ_2, the second best social value of green capital, only:

$$\dot{Y} = I(\zeta_2) - \delta Y$$
$$\dot{\zeta}_2 = (\rho + \delta)\zeta_2 - \phi\zeta_1(0)e^{\rho t}$$

Therefore, for a given level of the carbon tax $\widetilde{\tau}$, the solution is the same with the two instruments. ζ_2, the second best value of green capital, is the same in the two cases, and ζ_1, the second best carbon value, is the same as well. What differs are the private value of green capital, μ_d, and the present value of subsidies or FIP provided to electricity producers during the carbon era. This proves (ii).

As Y and ζ_2 are the same with the two instruments, $\mu_d^s + s = \mu_d^\sigma$, where the superscripts s and σ denote respectively the case with a subsidy and the case with a FIP. It follows that:

$$\begin{aligned}\dot{\mu}_d^s + \dot{s} = \dot{\mu}_d^\sigma &\Leftrightarrow (\rho + \delta)\mu_d^s - \phi\widetilde{\tau} + \dot{s} = (\rho + \delta)\mu_d^\sigma - \phi(\widetilde{\tau} + \sigma) \\ &\Leftrightarrow (\rho + \delta)\mu_d^s + \dot{s} = (\rho + \delta)(\mu_d^s + s) - \phi\sigma \\ &\Leftrightarrow \dot{s} = (\rho + \delta)s - \phi\sigma\end{aligned}$$

that is equation (31) in the text.

We now turn to the comparison of the present values of subsidies and FIP. When the carbon tax is "small", $\widetilde{\tau} \in [\underline{\tau}_1, \overline{\tau}]$, we are in the interior case ($s(t)$ and $\sigma(t) > 0$ $\forall t \in [0, \widetilde{T}]$) and the discounted present value of the subsidies to green investment provided over the carbon era is:

$$\int_0^{\widetilde{T}} e^{-\rho t} s(t) I(t) dt = \int_0^{\widetilde{T}} e^{-\rho t} s(t)(\dot{Y}(t) + \delta Y(t)) dt$$

By integration by parts,

$$\int_0^{\widetilde{T}} e^{-\rho t} s(t) \dot{Y}(t) dt = e^{-\rho \widetilde{T}} s(\widetilde{T}) Y(\widetilde{T}) - s(0) Y_0 - \int_0^{\widetilde{T}} e^{-\rho t} (\dot{s}(t) - \rho s(t)) Y(t) dt$$

Then, with $s(\widetilde{T}) = 0$,

$$\int_0^{\widetilde{T}} e^{-\rho t} s(t) I(t) dt = -s(0) Y_0 - \int_0^{\widetilde{T}} e^{-\rho t} (\dot{s}(t) - (\rho + \delta) s(t)) Y(t) dt$$

Finally, using equation (31) in the text we obtain (32). This proves (iii).

F Adding extraction costs and learning effects

The constant unit extraction cost is $c_x > 0$. The investment cost function is $C(I, Y)$, with $C_I > 0$ and $C_Y < 0$.

F.1 Decentralized equilibrium

The equilibrium seller's price for fossil resources is equal to the unit extraction cost:

$$p_x = c_x$$

The electricity producer receives two types of subsidies: the FIP σ on his electricity production from renewable sources ϕY in the carbon era, and the subsidy internalizing the learning by doing Σ on his green capacity Y in the carbon and clean eras. He takes the dependence of the investment cost to green capacity as given. He accumulates green capacity up to the point where

$$C_I(I, Y) = \mu_d$$

with

$$\dot{\mu}_d = (r + \delta)\mu_d - \phi p_e$$

The optimal investment function is $I(\mu_d, Y)$.

At equilibrium, μ_d evolves according to:

$$\begin{aligned}\dot{\mu}_d =&(\rho+\delta)\mu_d - \phi(c_x+\tau+\sigma) - \Sigma, \quad \text{with } x = e(c_x+\tau) - \phi Y \quad \text{in the carbon era}\\ \dot{\mu}_d =&(\rho+\delta)\mu_d - \phi u'(\phi Y) - \Sigma \quad \text{in the clean era}\end{aligned}$$

F.2 Optimum

Society's net surplus is now:

$$W = \int_0^\infty e^{-\rho t}\left[u(e(t)) - c_x x(t) - C(I(t), Y(t))\right] dt$$

The optimality conditions are modified as follows:

$$C_I(I, Y) = \mu$$

which gives the optimal investment function, $I(\mu, Y)$, and

$$\begin{aligned}\dot{\mu} =&(\rho+\delta)\mu + C_Y(I(\mu, Y), Y) - \phi(c_x+\lambda), \quad \text{with } x = e(c_x+\lambda) - \phi Y \quad \text{in the carbon era}\\ \dot{\mu} =&(\rho+\delta)\mu + C_Y(I(\mu, Y), Y) - \phi u'(\phi Y) \quad \text{in the clean era}\end{aligned}$$

The carbon era comes to an end at T such that $x(T) = 0$, $Y(T) = \frac{1}{\phi}e(c_x + \lambda(0)e^{\rho T})$ and the carbon budget is exhausted: $\int_0^T x(t)dt = \overline{X}$.

The steady state is defined by:

$$\begin{aligned}I(\mu, Y) &= \delta Y\\ (\rho+\delta)\mu &= \phi u'(\phi Y) - C_Y(I(\mu, Y), Y)\end{aligned}$$

F.3 Second best

F.3.1 Clean era

The regulator's program in the clean era reads:

$$\begin{aligned}&\max_{\sigma(.)} \int_{\widetilde{T}}^\infty e^{-\rho t}\left[u(\phi Y(t)) - C(I(\mu_d(t), Y(t)), Y(t))\right] dt\\ &\dot{Y}(t) = I(\mu_d(t), Y(t)), Y(t) - \delta Y(t)\\ &\dot{\mu}_d(t) = (\rho+\delta)\mu_d(t) - \phi u'(\phi Y(t) - \Sigma(t))\\ &Y(\widetilde{T}) \text{ given}\end{aligned}$$

The Hamiltonian is:

$$\mathcal{H} = u(\phi Y) - C\left(I(\mu_d, Y), Y\right) + \zeta_2 \left(I(\mu_d, Y) - \delta Y\right) + \zeta_3 \left[(\rho+\delta)\mu_d - \phi u'(\phi Y) - \Sigma\right]$$

The Hamiltonian is linear in Σ, and $\partial\mathcal{H}/\partial\Sigma = -\zeta_3$. Therefore we have:

$$\Sigma(t) \begin{cases} > 0 & \text{if} \quad \zeta_3 = 0 \\ = 0 & \text{if} \quad \zeta_3 > 0 \end{cases}$$

The other first order conditions are:

$$\begin{aligned} \dot{\zeta}_2 &= (\rho+\delta)\zeta_2 - \phi u'(\phi Y) + (C_I - \zeta_2) I_Y + C_Y + \zeta_3 \phi^2 u''(\phi Y) \\ \dot{\zeta}_3 &= -\delta\zeta_3 + (C_I - \zeta_2) I_{\mu_d} \end{aligned}$$

where C_I is the partial derivative of the cost function $C\left(I(\mu_d, Y), Y\right)$ with respect to its first argument, I, and C_Y the partial derivative of this same function with respect to its second argument, Y.

In the case where $\Sigma > 0$ over a non-degenerate interval of time, $\zeta_3 = \dot{\zeta}_3 = 0$ over this interval and the last FOC yields $C_I = \zeta_2$, implying that $\mu_d = \zeta_2$ (remember that the optimal investment of the power utility is given by $C_I = \mu_d$). Then, the comparison of the equations of evolution of μ_d and ζ_2 shows that:

$$\Sigma = -C_Y$$

F.3.2 Carbon era

Let us denote $\sigma^{tot} = \sigma + \Sigma/\phi$. For a given $\widetilde{\tau}$ chosen by the regulator, his programme reads:

$$\begin{aligned} &\max_{\sigma(.)} \int_0^{\widetilde{T}} e^{-\rho t} \left[u(e(c_x + \widetilde{\tau})) - c_x \left(e(c_x + \widetilde{\tau}) - \phi Y(t)\right) - C(I(\mu_d(t), Y(t)), Y(t))\right] dt \\ &\quad \dot{X}(t) = e(c_x + \widetilde{\tau}) - \phi Y(t) \\ &\quad \dot{Y}(t) = I(\mu_d(t), Y(t)) - \delta Y(t) \\ &\quad \dot{\mu}_d(t) = (\rho+\delta)\mu_d(t) - \phi(c_x + \widetilde{\tau} + \sigma^{tot}(t)) \\ &\quad X(t) \le \overline{X} \\ &\quad X(0) = 0 \ , \ Y(0) = Y_0 \ , \ Y(\widetilde{T}) = \frac{1}{\phi} e(c_x + \widetilde{\tau}), \ \mu_d(0) \ \text{ free} \end{aligned}$$

The associated Hamiltonian is:

$$\begin{aligned}\mathcal{H} =& u(e(c_x + \widetilde{\tau})) - c_x \left(e(c_x + \widetilde{\tau}) - \phi Y\right) - C(I(\mu_d, Y), Y) \\ & - \zeta_1(e(c_x + \widetilde{\tau}) - \phi Y) + \zeta_2 \left(I(\mu_d, Y) - \delta Y\right) + \zeta_3 \left[(\rho + \delta)\mu_d - \phi(c_x + \widetilde{\tau} + \sigma^{tot})\right]\end{aligned}$$

The Hamiltonian is linear in σ^{tot}. The other first order conditions are:

$$\begin{aligned}\dot{\zeta}_1 &= \rho\zeta_1 \\ \dot{\zeta}_2 &= (\rho + \delta)\zeta_2 - \phi(c_x + \zeta_1) + (C_I - \zeta_2)I_Y + C_Y \\ \dot{\zeta}_3 &= -\delta\zeta_3 + (C_I - \zeta_2)I_{\mu_d}\end{aligned}$$

As $\sigma^{tot}(t) > 0 \ \forall t \in [0, \widetilde{T}]$ (the learning by doing component is necessarily positive), $\zeta_3 = 0$, the social and private values of green capital are equal, and:

$$\sigma^{tot} = \zeta_1 - \widetilde{\tau} - \frac{1}{\phi}C_Y$$